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SCHOOL OF ENGINEERING
OLD DOMINION UNIVERSITY
NORFOLK, VIRGINIA

Technical Report 75-M3

STUDIES IN FINITE ELEMENT ANALYSIS OF COMPOSITE MATERIAL STRUCTURES

By

Dale O. Douglas

Donna E. Holzmacher

Zoa C. Lane

and

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Final Technical Report

Prepared for the
National Aeronautics and Space Administration
Langley Research Center
Hampton, Virginia

Under
Grant NSG 1043
June 1, 1974 - May 31, 1975

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NOTICE

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Dr. J.G. Davis, Jr., Technical Monitor
Materials Division
Composites Section

Submitted by the
Old Dominion University Research Foundation
Norfolk, Virginia

September 1975

FOREWORD

This report presents four papers resulting from research conducted under a grant from NASA to the Old Dominion University Research Foundation entitled: "A Research Participation Program for Minority Engineering Students". The three undergraduate engineering students, Dale O. Douglas, Donna E. Holzmacher, and Zoa C. Lane, worked under the direction of Dr. Earl A. Thornton, Associate Professor of Mechanical Engineering and Mechanics.

The student-faculty team began their research in analysis of composite materials at Langley Research Center during a ten-week period in the summer of 1974. The work was continued during the academic year 1974-1975 at Old Dominion University.

Dr. John G. Davis, Jr., of the Composites Section, Materials Application Branch of the Materials Division served as technical monitor for the program. For his cooperation, encouragement, and counsel the authors express their deepest appreciation.

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FINITE ELEMENT ANALYSIS OF A PICTURE FRAME SHEAR TEST

By

Dale O. Douglas

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INTRODUCTION

Shear testing of composite materials is generally concerned with two principal areas of interest: (1) to determine the in-plane shear properties, or (2) to determine the interlaminar or normal shear properties. In-plane shear properties of a laminate are among the most difficult to determine because of problems in applying a state of uniform shearing stress. Concepts for determining in-plane shear properties include torsion tube tests, rail shear tests, and picture frame shear tests

The most direct method of applying pure shear is by torsion of a tube. This test method has proven to be a reliable means of determining in-plane shear properties (ref. 1). However fabrication techniques for high quality ± 45° metal matrix composite tubes have not yet been established. The difficultv of fabricating high quality tubes has stimulated research in other methods of shear testing.

Another type of shear test is the rail shear test. It uses a thin laminate, loaded along its length by two pairs of rails, leaving an unsupported central test section

In the present study an analysis of a picture frame shear test performed at Langley Research Center is presented. The purposes of the study were to determine the stress distributions in the picture frame shear test specimen and to determine the effect of local reinforcements on the stress distributions.

DESCRIPTION OF TEST

The experimental setup for a picture frame shear test is shown in figure 1. The picture frame shear test was used to

produce in-plane shear stress in the test panel. The shear panel was bonded to a frame constructed from four 1 in. x 1 in. steel edge bars designed to simulate fully clamped edge conditions. The panel specimen was bolted to a test frame by 0.375-in.-diameter bolts, seven per side. At each corner of the test frame, loads were applied to the pin joints by the testing machine. Tensile loads were applied to the vertical pins, and compressive loads were applied to the horizontal pins to produce shear loading in the test specimen.

TEST SPECIMEN

The test specimens were made using 7 in. x 7 in. borsic aluminum sandwich shear panels. With the addition of 1 in. x 1 in. steel edge bars, the overall dimensions of the shear panel specimen were 9 in. x 9 in. with a nominal thickness of 1 in. To permit installation of the pins on the test frame, a portion of the shear panel was cut away at each corner. Each corner had a radius of 0.25 in. The test specimen is shown schematically in figure 2.

The sandwich panel consisted of two face sheets separated by a honeycomb core. On each face sheet there were four plies (0.0285 in. thick) at a ± 45° layup. The panel face sheets were cut from 10-in.-square laminates. The filaments of the laminate were parallel to the applied loads. Some of the test specimens were reinforced with titanium doublers (0.060 in. thick) in the vicinity of the corner radii.

ANALYSIS OF SHEAR TEST

Finite element analyses were made to determine the in-plane stress distributions in the shear panel. The finite models represented the shear panel specimens using orthotropic, two-dimensional plane stress elements. Two general purpose finite element computer programs were utilized in the analysis of the shear panel. The first was NASTRAN (NASA Structural Analysis Program) which was

used on the CDC-6600 computer at Langley Research Center. NASTRAN (ref. 2) is a general purpose digital computer program for the analysis of large complex structures. The second program, SAP IV (Structural Analysis Program), was executed on an IBM-370, Model 158 computer at Virginia Polytechnic Institute & State University through the computer center at Old Dominion University. SAP IV (ref. 3) is a structural analysis program for static and dynamic response of linear systems. Symmetry of loading, geometry, and material properties made the analysis of only one quarter of the specimen sufficient.

NASTRAN embodied a finite element approach, wherein the distributed physical properties of the shear specimen were represented by a model (fig. 3) consisting of 490 membrane elements that were interconnected at 529 grid points. The grid point definition formed the basic framework for the structural model. All other parts of the structural model were referenced either directly or indirectly to the grid points. Each grid point had two degrees of freedom, the in-plane displacements. The elements used in the analysis were the quadrilateral membrane element CQDMEM and the triangular membrane element CTRMEM.

The steel edge bars of the test specimen were represented in NASTRAN as rigid boundaries. The rigid boundaries were modeled using multipoint constraints in the NASTRAN program The constraints were applied to grid points on the test frame edge of the finite element model so that these points deformed as a straight line. Static loads were applied to the structural model through nodes constrained to the rigid boundary.

The loads were from Langley Research Center Test 560, Run 7; a horizontal load of 5004.9 lb and a vertical load of 5039.4 lb are shown in figure 3 at the points of application.

SAP embodied a finite element approach where the shear specimen was represented by a model (fig. 4) consisting of 554 membrane elements that were interconnected at 595 nodal points.

The steel edge bars of the test specimen were represented in SAP as deformable boundaries. The deformable boundaries were simulated by the addition of 64 plane stress membrane elements to the NASTRAN model. The horizontal and vertical applied loads were represented by statically equivalent loads applied along the simulated boundary. Nine colinear loads were applied at nodal points nearest the center of each bolt hole. These loads were applied at an angle of 45 degrees. The magnitudes of these applied loads are given in figure 4. Stresses were computed at the centroid of each element using the stress print option available in SAP.

The titanium doublers used for local reinforcement at corner radii were modeled with an addition of 21 finite elements on existing elements at the extreme corner of the sandwich panel. The material elasticity matrix for titanium and borsic aluminum is given in table 1.

Table 1. Material elasticity matrix.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{12} & G_{22} & G_{23} \\ G_{13} & G_{23} & G_{33} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_x \\ \gamma_{xy} \end{Bmatrix}, \text{ psi}$$

	G_{11}	G_{12}	G_{13}	G_{22}	G_{23}	G_{33}
Borsic Aluminum	2.81E+7	5.65E+6	0	2.81E+7	0	9.5E+6
Titanium	1.81E+7	6.15E+6	0	1.81E+7	0	6.15E+7

The NASTRAN finite element model of the shear panel, simulating a rigid boundary, had 1000 degrees of freedom. Using a CDC-6400 computer, it took 945 CPU seconds for the program to execute. In contrast to the NASTRAN model, the SAP finite model

had 1132 degrees of freedom with a bandwidth of 1106. Due to the excessive storage required by the large bandwidth, the SAP finite element program was unable to execute. To optimize the bandwidth, the nodes were renumbered using a computer program, BANSAP. With this renumbering, the SAP program had a final bandwidth of 69. It then completed execution in 160 CPU seconds.

RESULTS AND DISCUSSION

The stresses computed in the shear panel for the loads applied to the rigid (NASTRAN) and deformable (SAP) boundary models are given in figures 5 through 8. Normal stresses σ_x and σ_y are plotted as ordinates with the horizontal and vertical coordinates from the center of the shear panel as abscissae.

The stress distributions along the horizontal and vertical axes of both the rigid and deformable boundary models are uniform near the center of the specimen. The uniform stress values differ considerably between the two models; the uniform normal stresses predicted by the rigid boundary model are nearly three times the stresses predicted assuming a deformable boundary. These results indicate that the assumption of a rigid boundary should not be made.

There is an appreciable stress concentration at the corner fillets. The stress components perpendicular to the lines of symmetry rise sharply at the corners. For example, figure 7 shows that in the deformable boundary model the stress component σ_y increases from 10 ksi to about 105 ksi indicating a stress concentration factor of over 10.

Contour plots of the principal shearing stresses for the rigid and deformable boundary models are shown in figures 9 and 10. The shearing stresses are uniform only over a small portion of the specimen. Figure 10 shows that the shearing stress may vary by as much as 25 percent over the center one-half of the specimen.

The effect of the reinforcing titanium doubler on the normal stresses σ_x and σ_y is shown in figures 11 through 14. These results indicate that the doubler significantly reduced the stresses along the x-axis near the fillet. The critical stress σ_y on this axis was reduced by about one-half. However, stress distributions along the vertical axis and in the center of the shear panel show no reinforcing effects of the titanium doubler.

The contours of the principal shearing stress in the specimen with the titanium doubler are shown in figure 15. By comparing this figure with figure 10 it can be seen that the doubler tended to reduce the region of nearly uniform shearing stress since the contours in figure 15 are closer to the center of the panel. As expected there is also an appreciable local disturbance in the shearing stresses in the vicinity of the doubler.

CONCLUDING REMARKS

Two finite element analyses of a picture frame shear test of a borsic aluminum test specimen have been performed. Two methods for modeling the specimen test frame have been investigated. Results for nominal stresses and principal shear stress have been presented for Test 560, Run 7 conducted at Langley Research Center.

There were striking differences in the stress distributions predicted by the rigid (NASTRAN) and deformable (SAP) boundary models. It was found that it is not realistic to assume the test fixture to be a rigid frame. In the regions of nearly uniform stress, the stresses predicted by the deformable boundary models were approximately one third of the stresses predicted by the rigid boundary model. In the vicinity of the corner, the stresses predicted by the two models nearly coincided.

The constant principal shear stress, τ_{max} was uniform over only a very small region in the center of the shear panel specimen. Moreover, at the corners near the fillets, there were steep gradients with stresses being highly concentrated.

The effect of a local reinforcing titanium doubler has been evaluated. It was found that the doubler reduced the maximum nominal stress in the vicinity of the fillet by about 50 percent.

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2. McCormick, C.W., Ed., The NASTRAN Users' Manuals, NASA, June 1972.

3. Wilson, E.L., Peterson, F.E., and Bathe, K., SAP IV: A Structural Analysis Program for Static and Dynamic Response of Linear Systems, Report No. EERC 73-11, Engineering Analysis Corporation, Berkeley, California, June 1973.

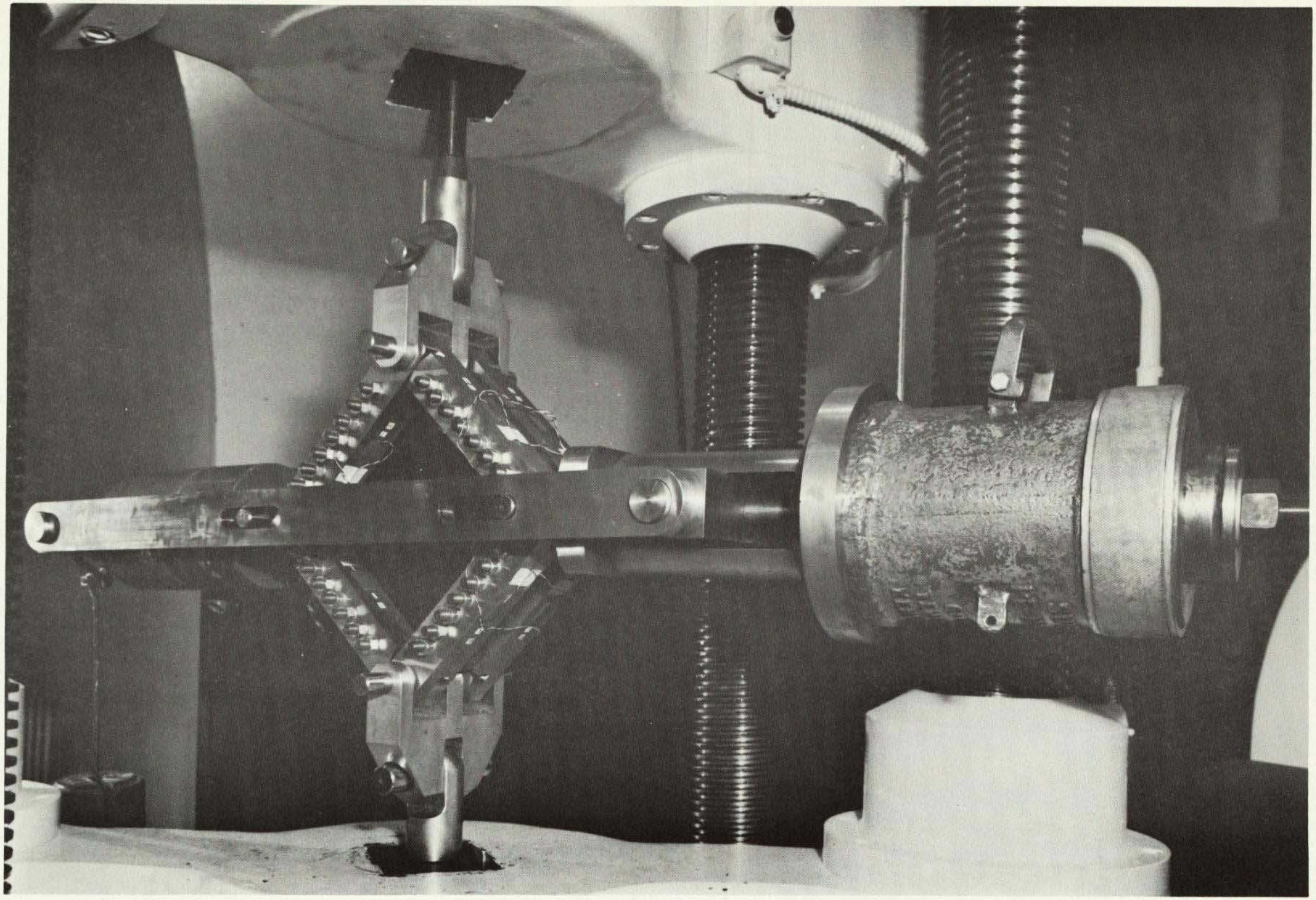

Figure 1. Picture Frame Shear Test Experimental Setup at Langley Research Center.

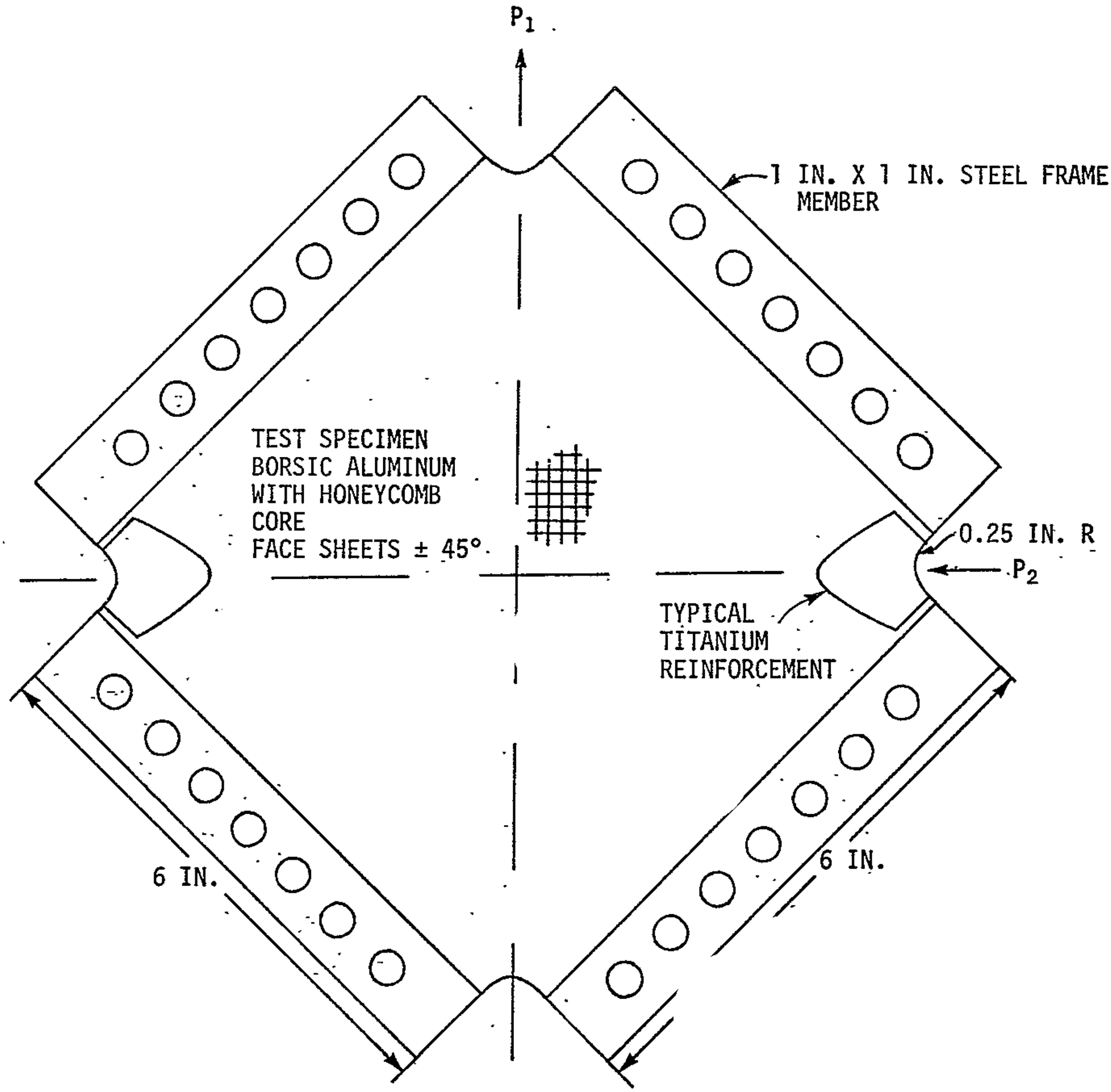

Figure 2. Schematic of Test Specimen.

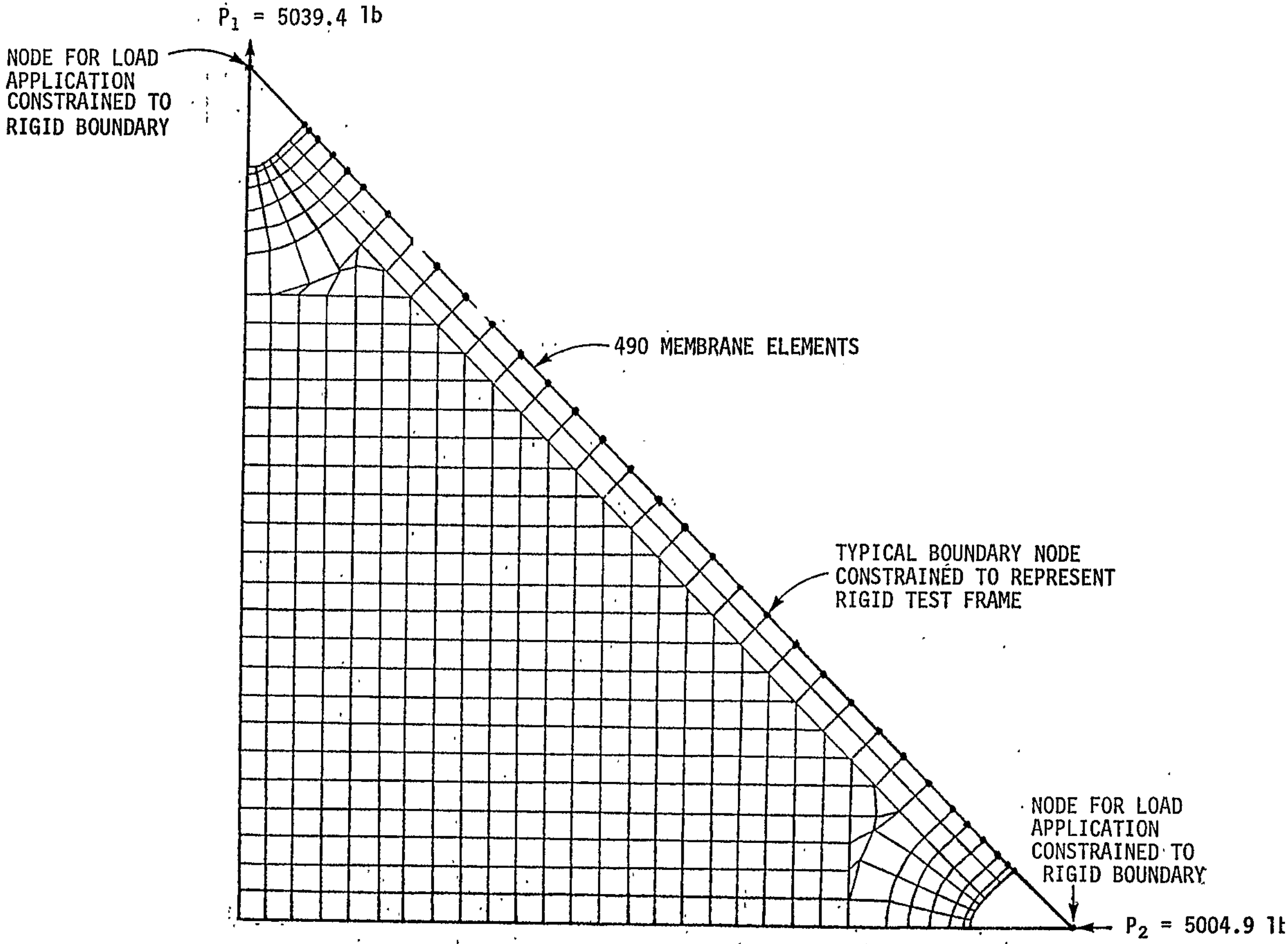

Figure 3. NASTRAN Finite Element Model of Shear Panel with Rigid Boundary.

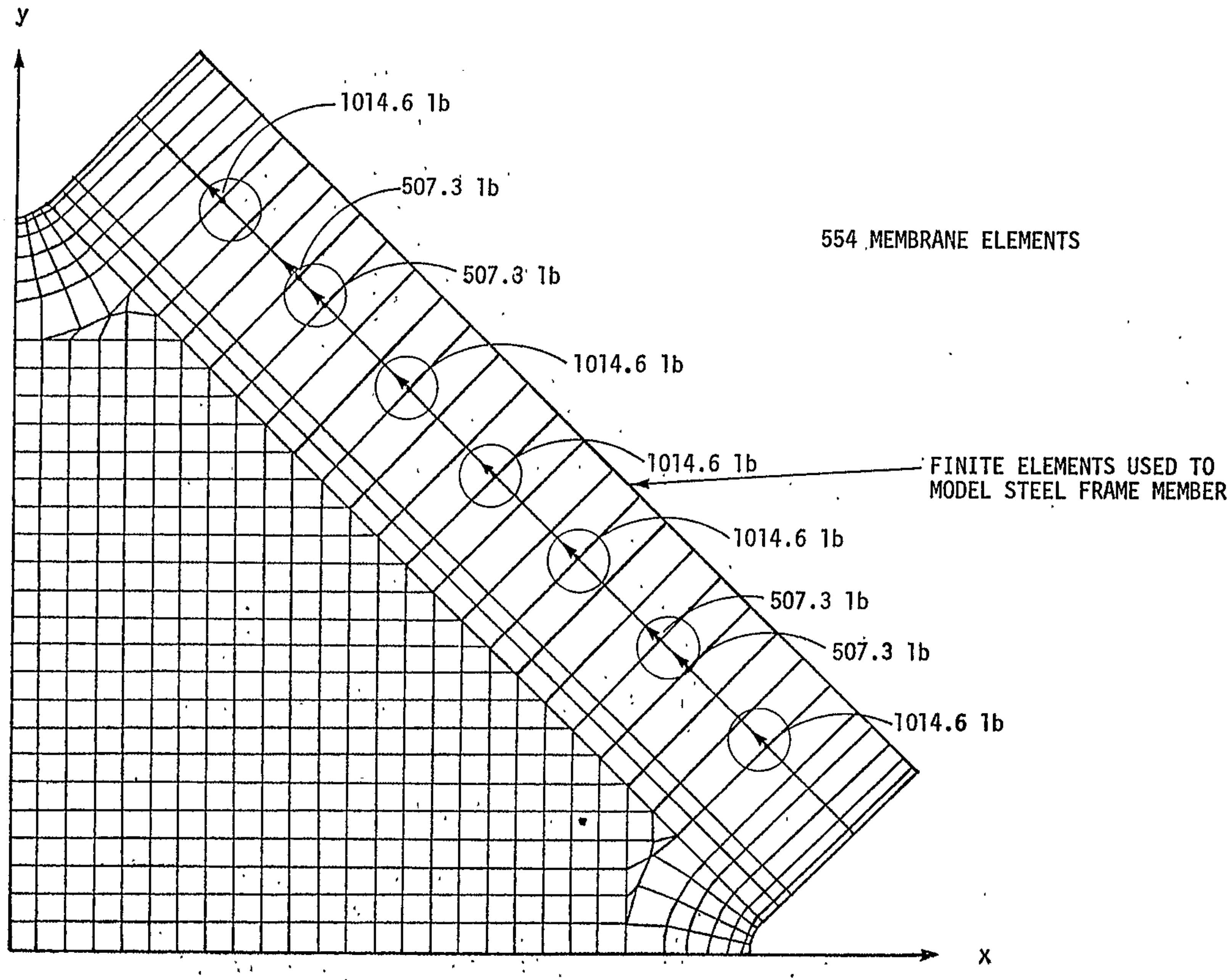

Figure 4. SAP Finite Element Model of Shear Panel (Deformable Boundary).

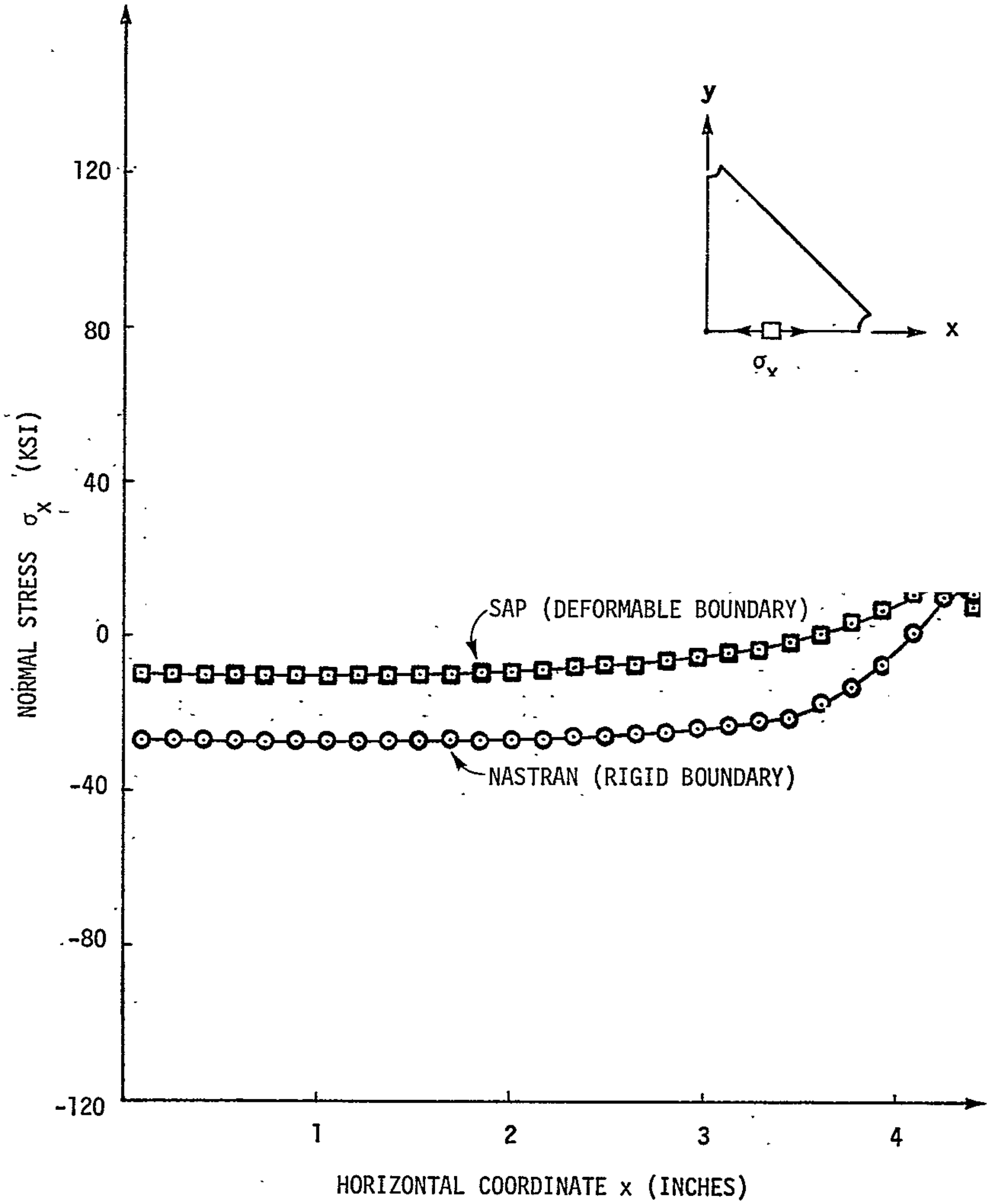

Figure 5. Normal Stress σ_x as a Function of Horizontal Coordinate Along Center Line of Shear Panel Specimen

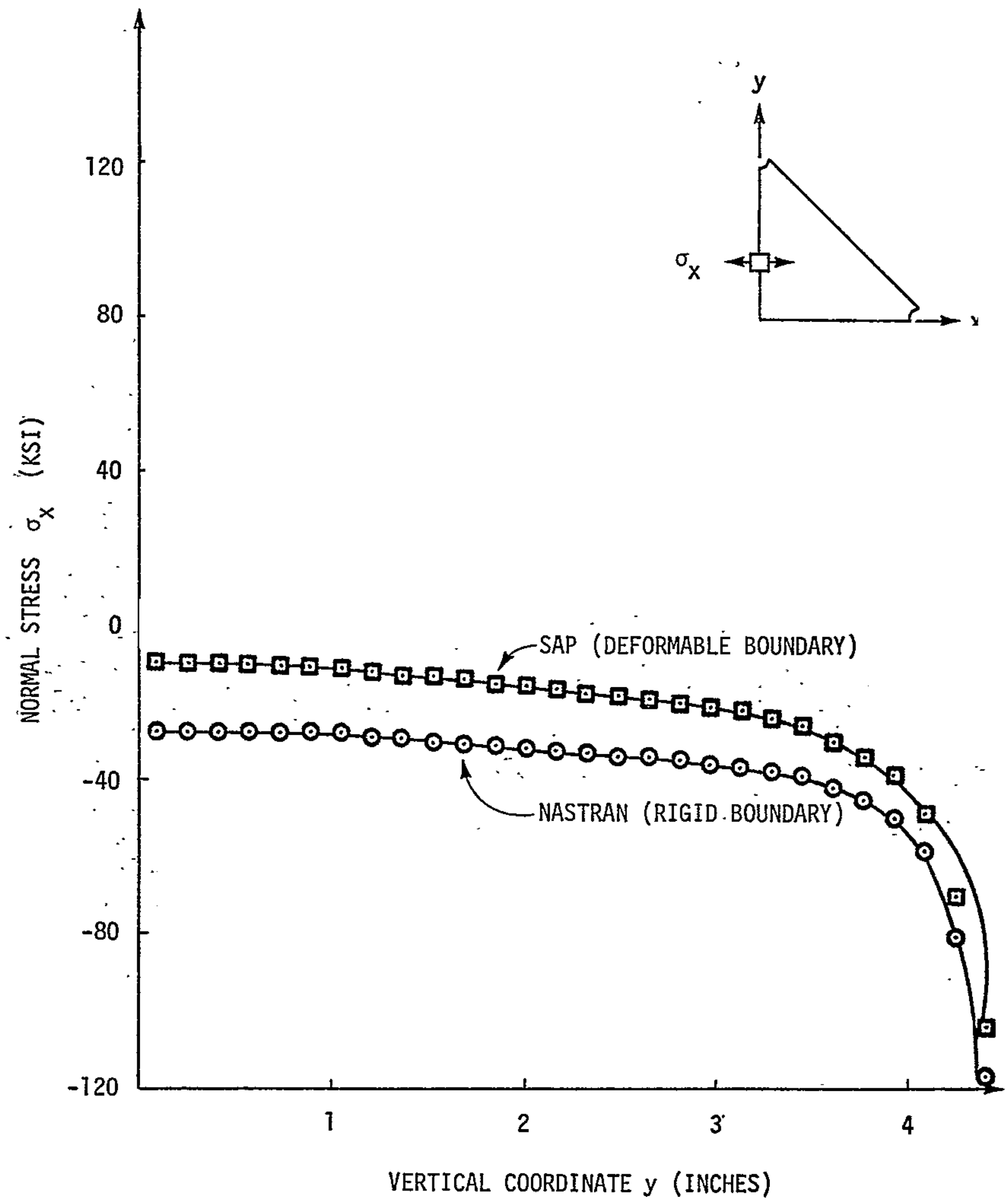

Figure 6. Normal Stress σ_x as a Function of Vertical Coordinate Along Center Line of Shear Panel Specimen.

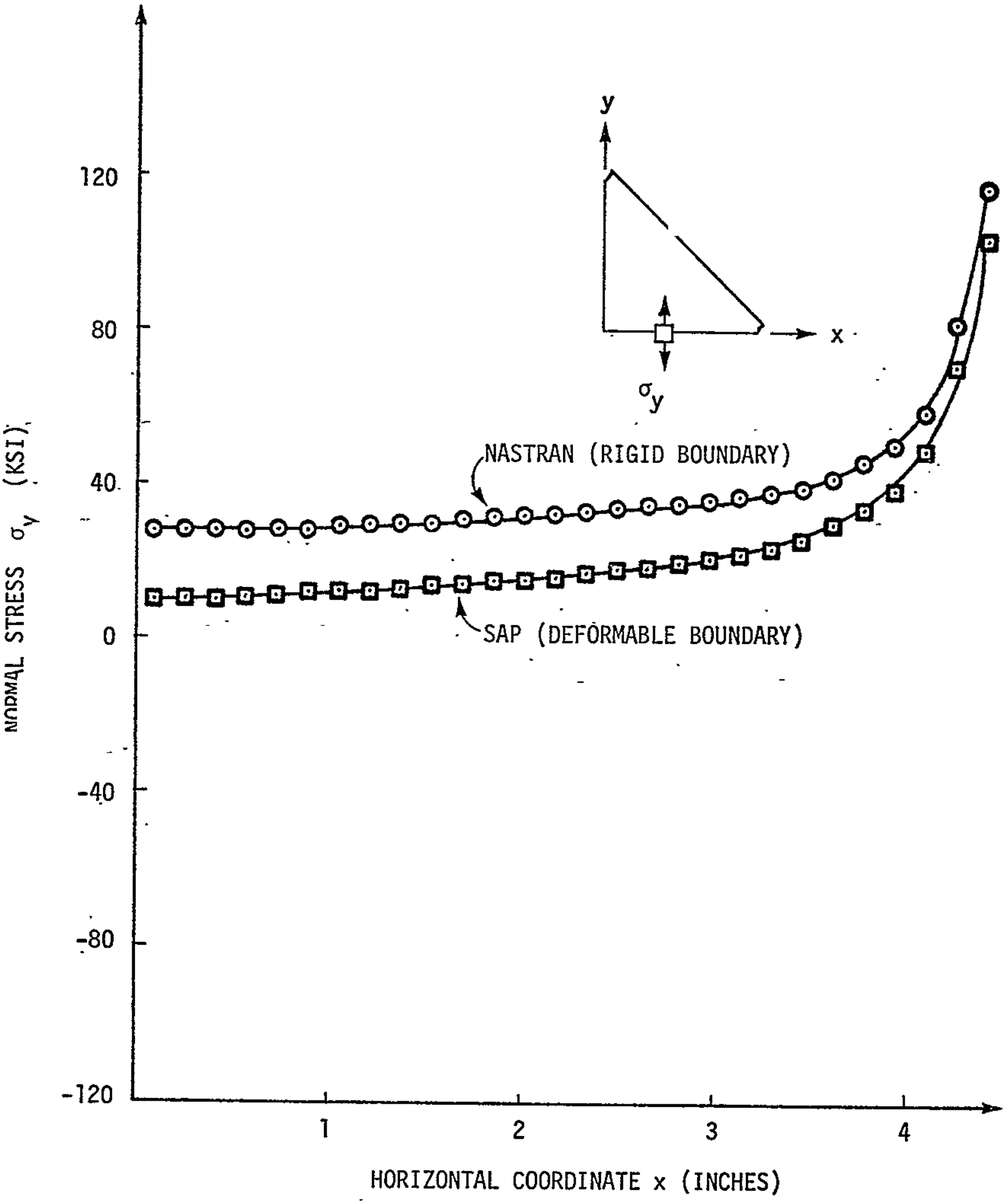

Figure 7. Normal Stress σ_y as a Function of Horizontal Coordinate Along Center Line of Shear Panel Specimen.

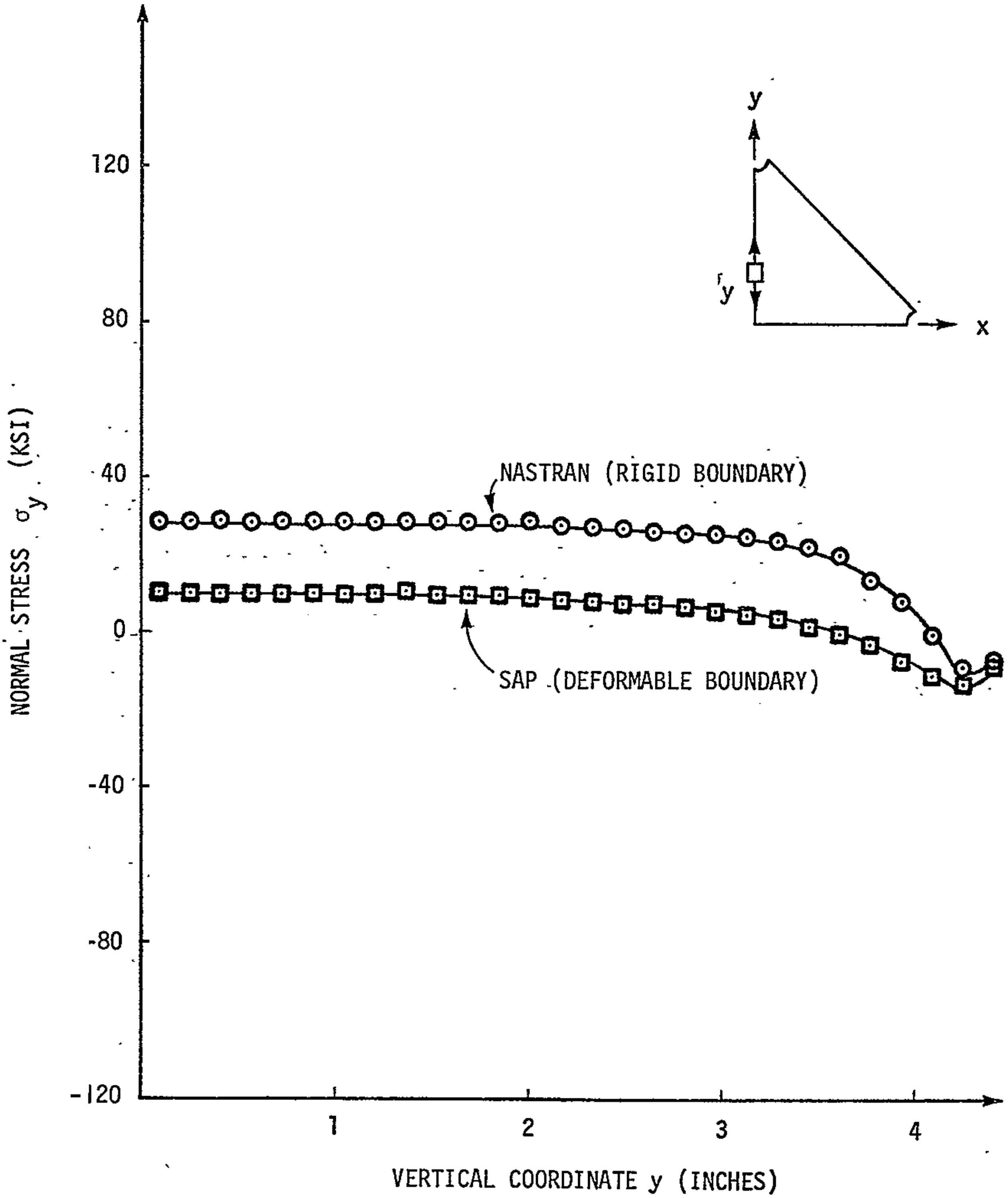

Figure 8. Normal Stress σ_y as a Function of Vertical Coordinate Along Center Line of Shear Panel Specimen.

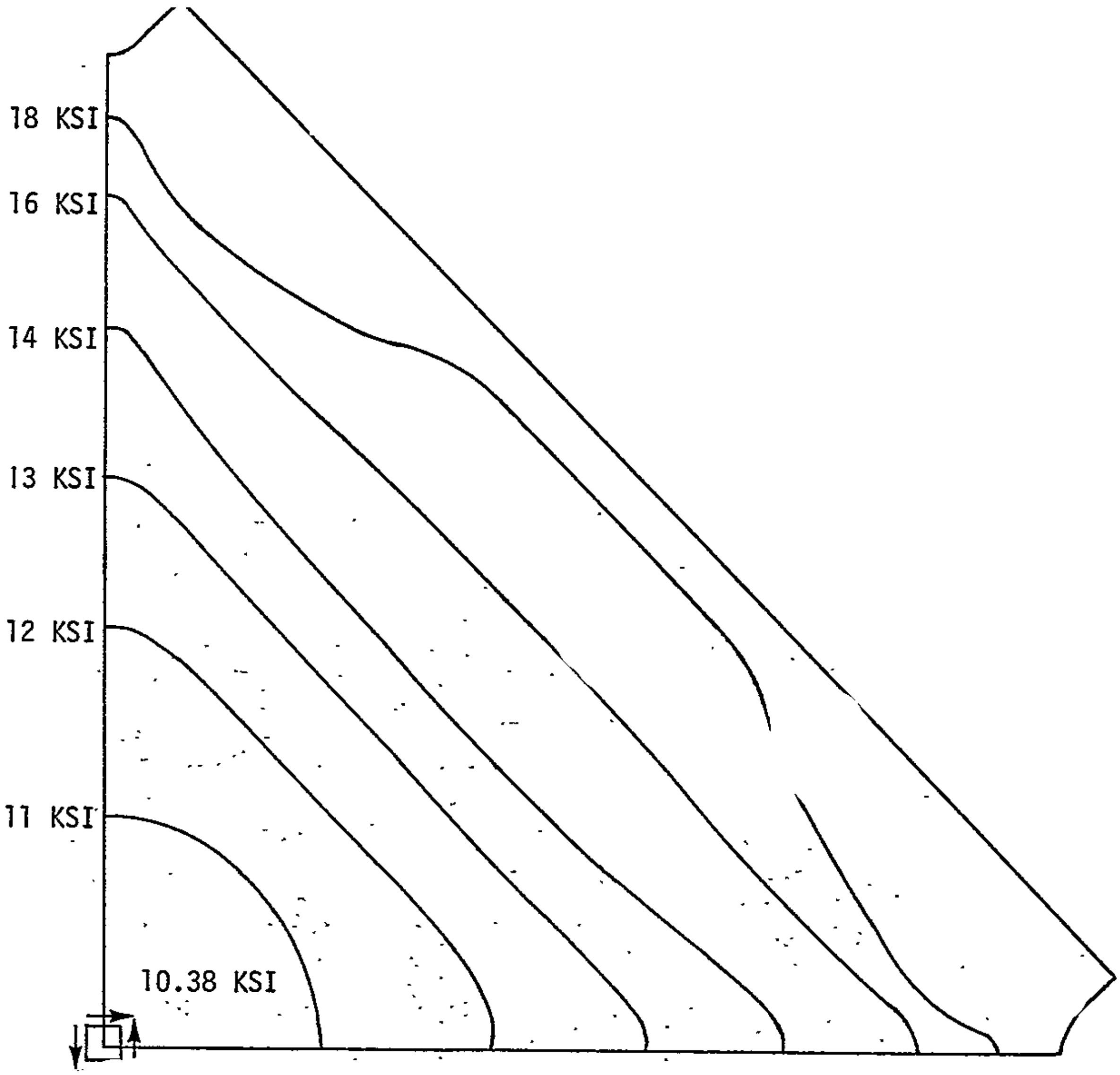

Figure 9. Contours of Constant Principal Shear Stress, τ_{max}, Predicted by NASTRAN (Rigid Boundary).

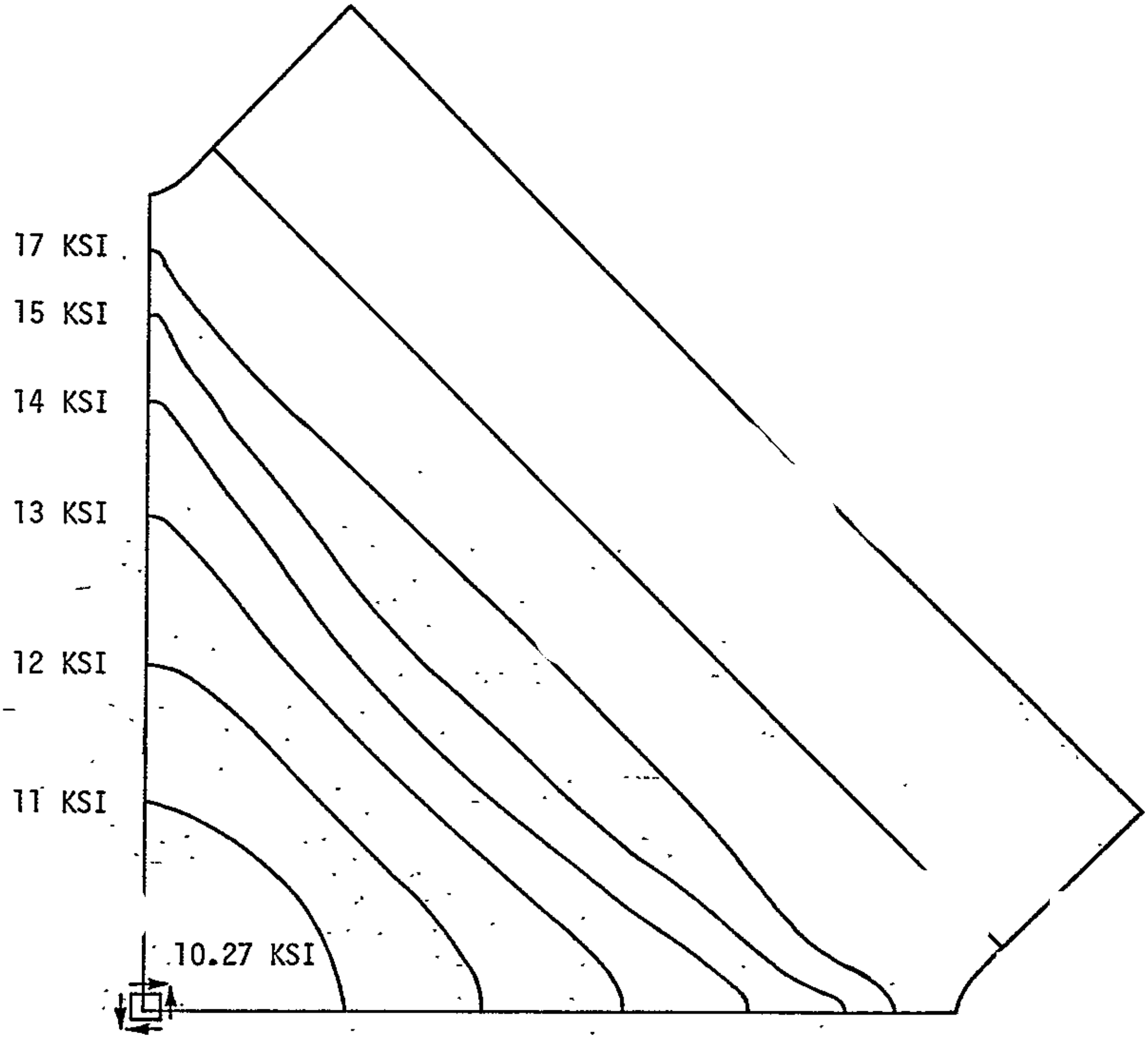

Figure 10. Contours of Constant Principal Shear Stress, τ_{max}, Predicted by SAP (Deformable Boundary).

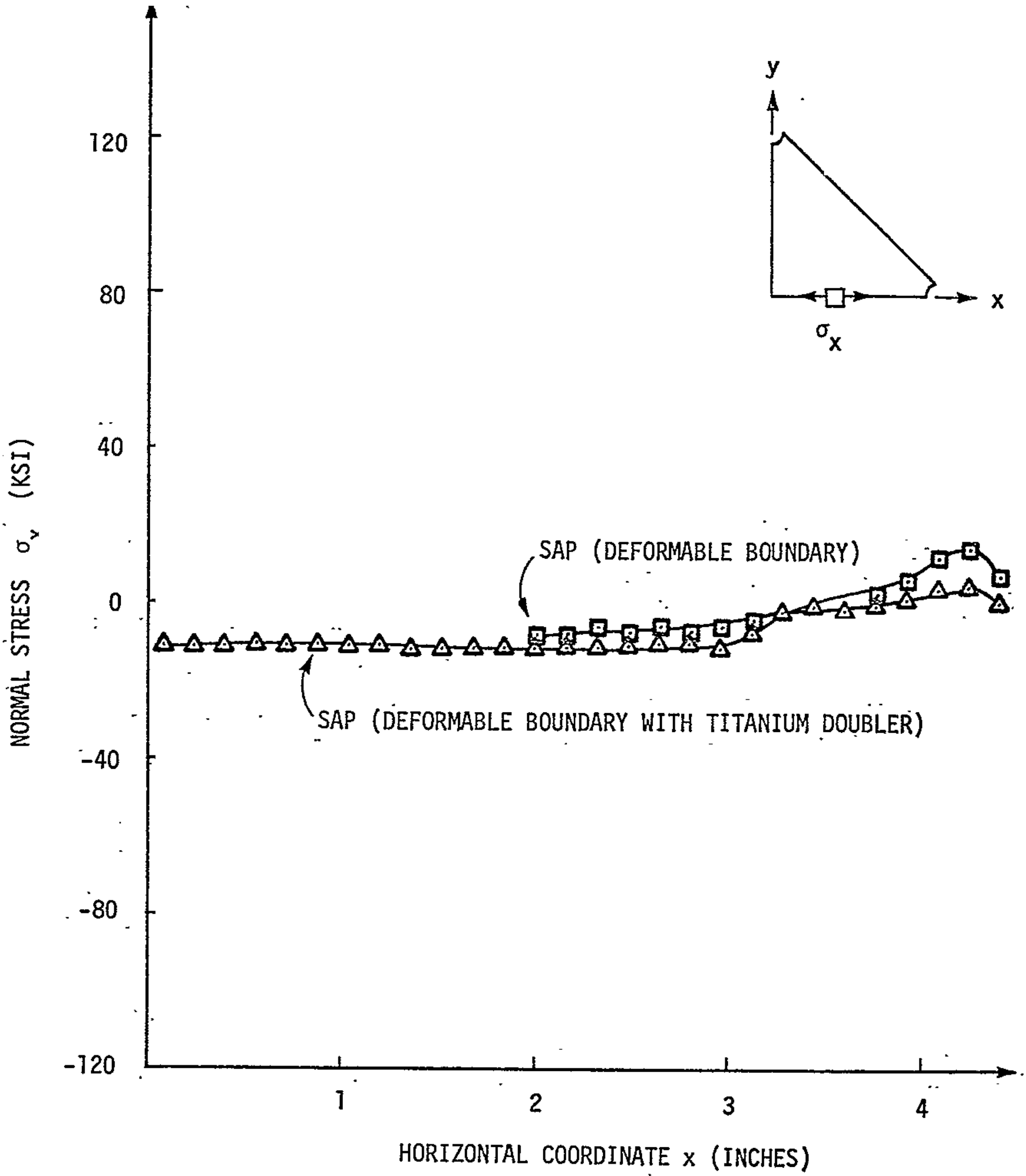

Figure 11. Normal Stress σ_x as a Function of Horizontal Coordinate Along Center Line of Shear Panel Specimen.

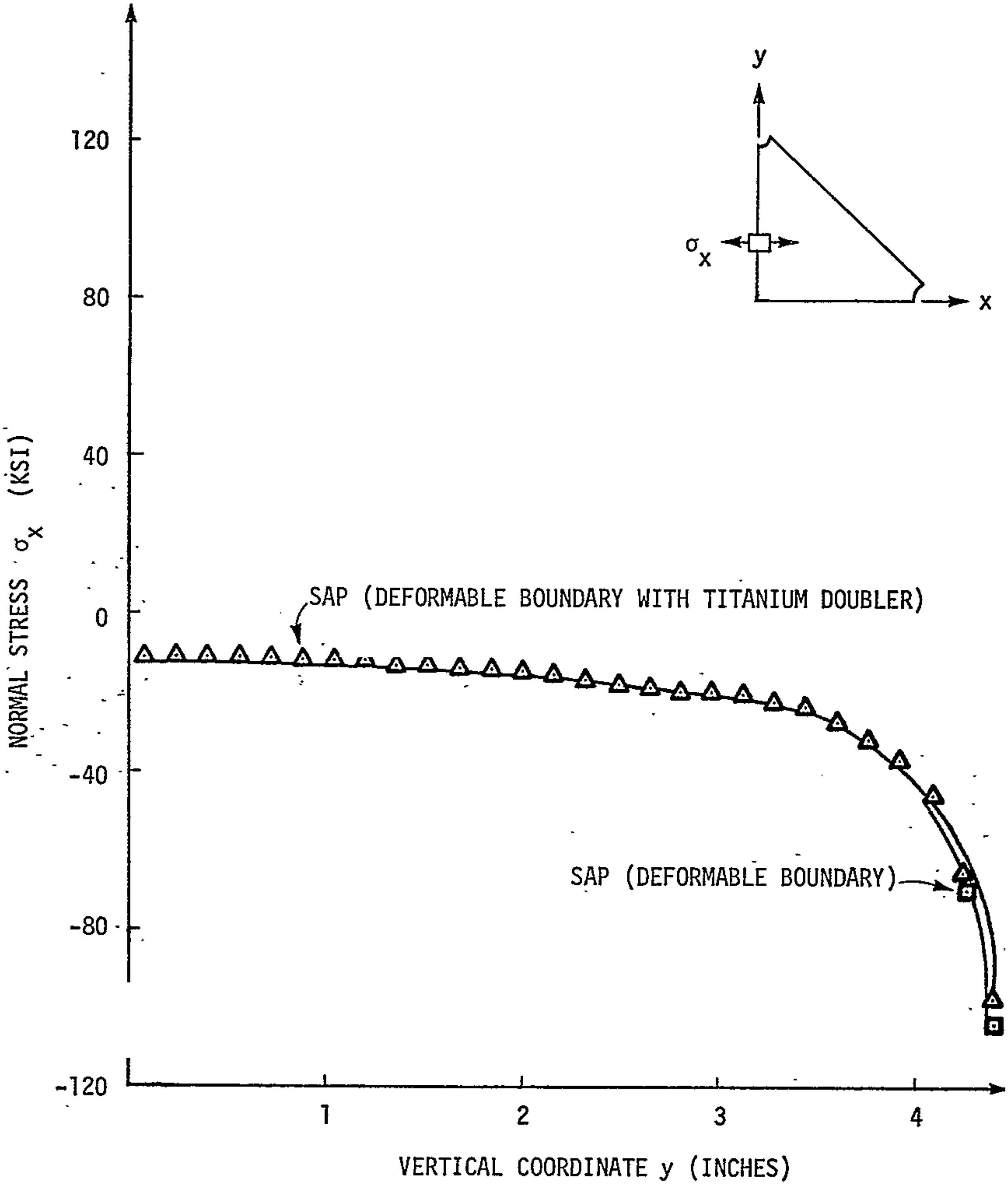

Figure 12. Normal Stress σ_x as a Function of Vertical Coordinate Along Center Line of Shear Panel Specimen.

the centroidal coordinate of the finite elements bordering the hole. Comparison of figures 6 through 8 and figure 10 shows that qualitatively the finite element analysis of the anistropic composite and the isotropic elasticity solution are in close agreement. This agreement serves to validate the finite element solution.

The variation of the longitudinal membrane force in an isotropic infinite medium is shown in figure 11 in terms of the x coordinate of the composite specimen to facilitate comparison with the finite element solution given in figure 9. The elasticity solution shows an extremely sharp gradient for the membrane force in the vicinity of the hole. This sharp variation raises questions about the accuracy of the finite element solution in this region. Since the NASTRAN finite element assumed constant stress within the element, it is possible that the peak stress was underestimated because not enough elements were used to accurately represent the stress gradient. The variation of the stress away from the hole according to the isotropic solution shows that in a distance of about five radii (5a = 0.48 in.) away from the hole the force has decreased to one-tenth of its maximum value. This result supports the findings of figures 6 through 8 in which the membrane force distributions in the center and outside holes were very nearly the same. This occurred because there were no hole interaction effects since the holes were more than five radii apart. Only very small edge effects were present for the same reason.

CONCLUDING REMARKS

A finite element analysis of an extra graph[illegible] bolted joint specimen has been performed. Two methods were used to represent bolt transfer loads. The first method assumed a perfect fit and modeled the bolt loading as a cosine distributon over one-half of the boundary of the hole. The second method assumed an imperfect fit and used a nonlinear computer analysis to determine the contact area and bolt transfer loads. The

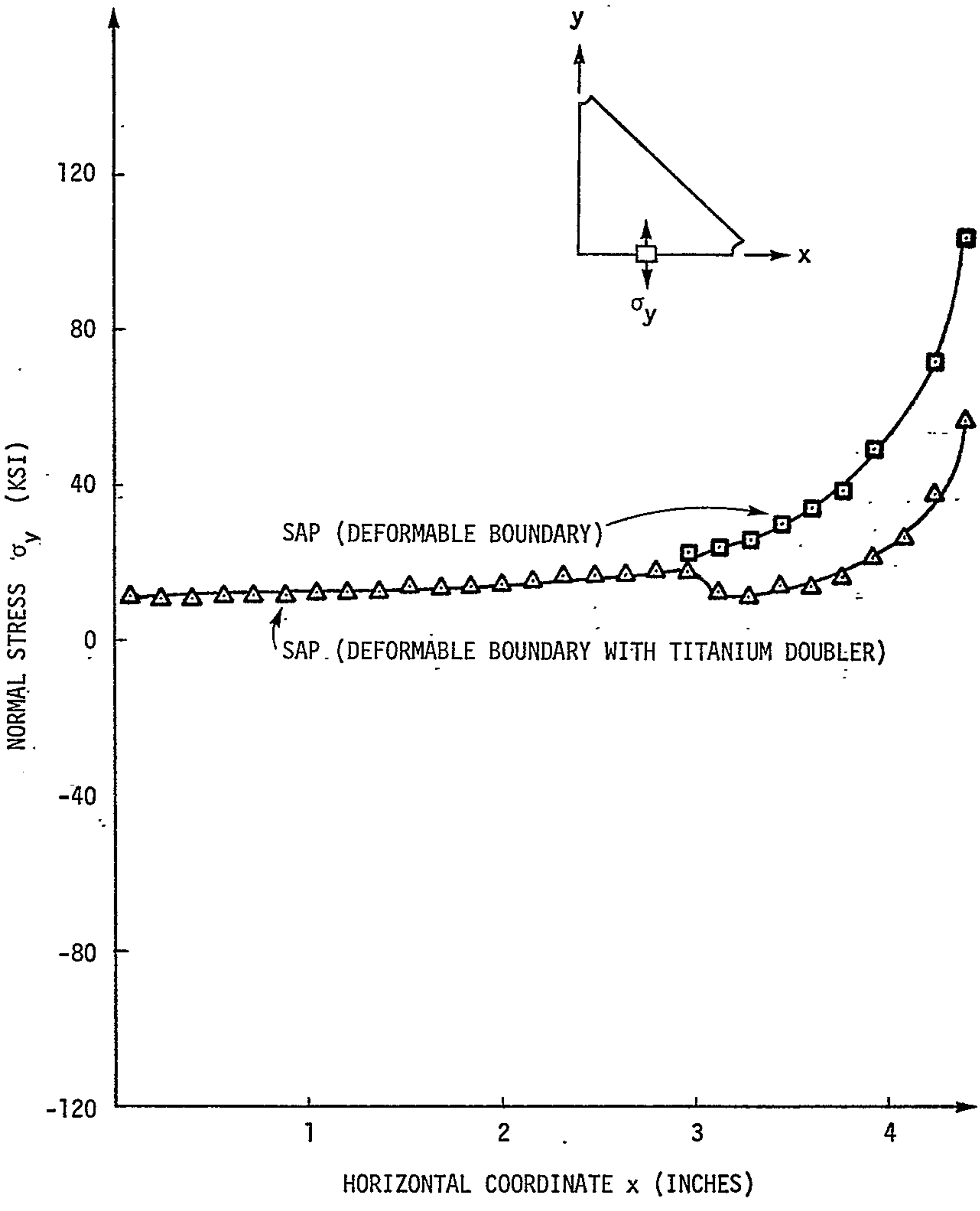

Figure 13. Normal Stress σ_y as a Function of Horizontal Coordinate Along Center Line of Shear Panel Specimen.

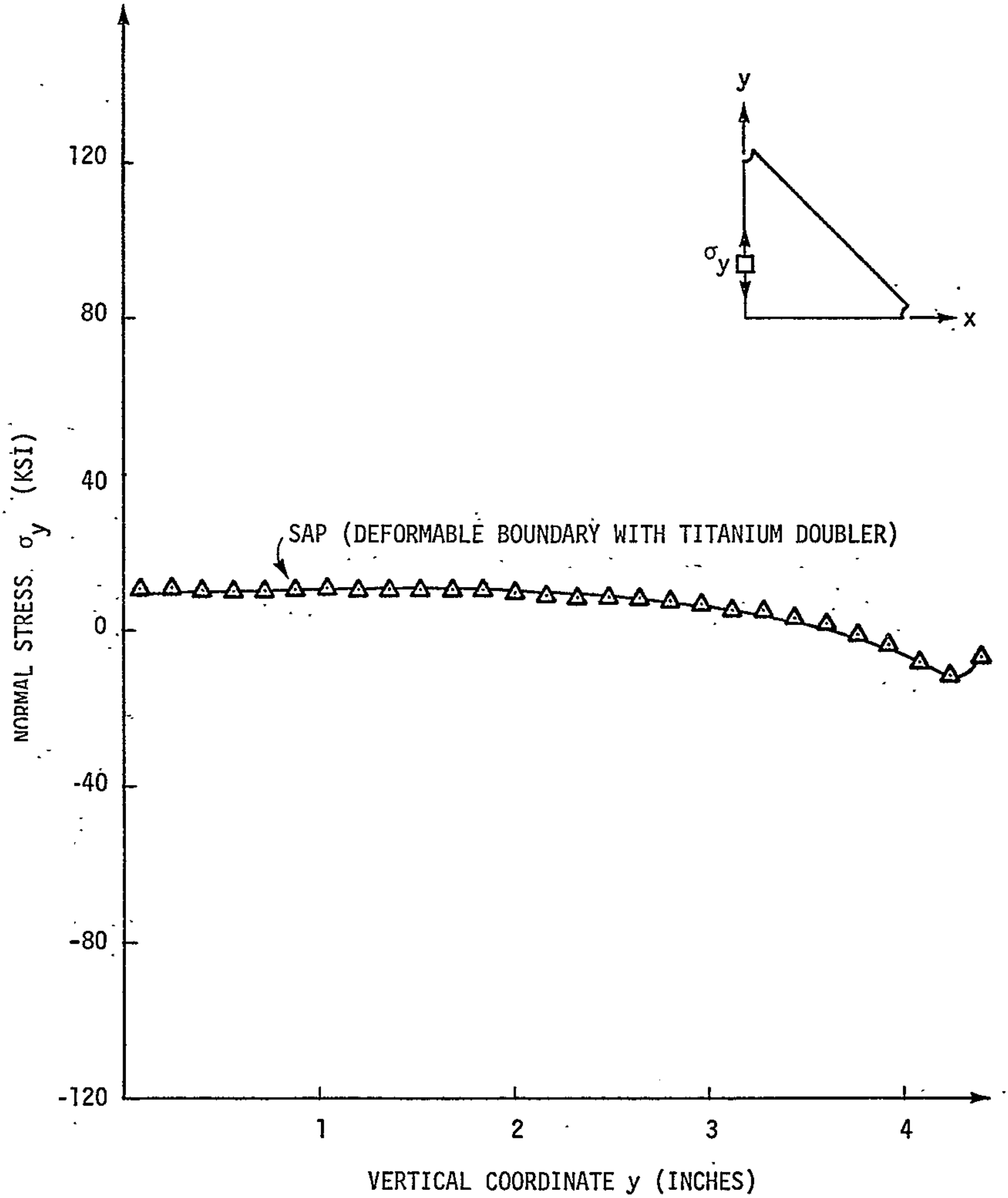

Figure 14. Normal Stress σ_y as a Function of Vertical Coordinate Along Center Line of Shear Panel Specimen.

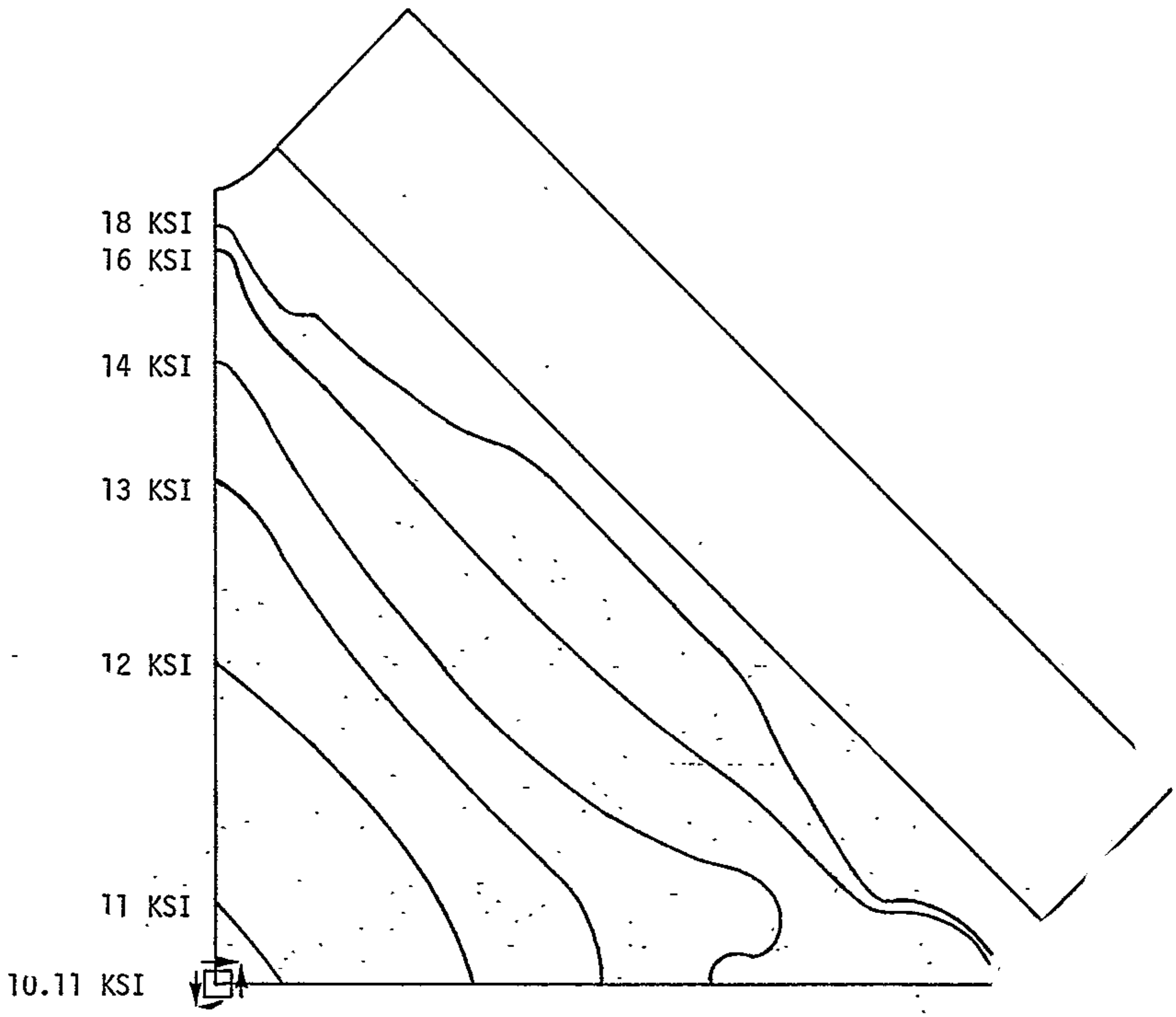

Figure 15. Contours of Constant Principal Shear Stress, τ_{max}, Predicted by SAP (Deformable Boundary with Reinforcing Titanium Doubler).

BANSAP: A BANDWIDTH REDUCTION PROGRAM FOR SAP IV

By

Donna E. Holzmacher

BANSAP: A BANDWIDTH REDUCTION PROGRAM FOR SAP IV

By

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INTRODUCTION

For analysis, a structure may be broken down into parts known as finite elements. The elements of the structure may be one-dimensional such as a rod, two-dimensional such as a triangle, or quadrilateral, or three-dimensional such as a parallelepiped. The elements are positioned and described by nodes which, when connected, describe the structure. Static analysis using finite elements is accomplished by solving simultaneous equations. These equations when written in matrix form are characterized by banded coefficient matrices. Computer time and storage can be saved if the bandwidth of the matrix is a minimum. This occurs with adept numbering of the nodal points of the structure. If the nodes are numbered in an optimum way the non-zero values in the matrix will lie in a band about the diagonal. The bandwidth of a matrix is defined here as the maximum difference between any two connected nodes plus one to take into account the diagonal term.

As a particular example, consider the plane structure shown in figure 1. The displacements of this structure are determined by solving

$$(K)\ \{U\} = \{P\}$$

where (K) is the stiffness matrix, {U} is the displacement vector, and {P} is the load vector. The (K) matrix is arranged according to the connectivity of the nodes 1 through 9 of the triangular elements. The connectivity matrix for the above structure is represented in figure 2 showing that node 1 is connected to nodes 2, 8, and 9, and node 2 is connected to

nodes 1, 2, 3, 4, and 9, etc. The actual values in the stiffness matrix, corresponding to the positions of the matrix above, depend on the geometry and material of the structure.

The bandwidth of the connectivity matrix shown in figure 2 is 9. If the nodes are renumbered as in figure 3, the corresponding connectivity matrix as shown in figure 4 has a new, reduced bandwidth of 4.

In order to efficiently renumber the nodes of structures for finite element analysis a number of algorithms have been developed and incorporated into bandwidth reduction programs. Prior to 1969, authors who developed techniques to reduce the bandwidths of matrices included Always and Martin, Tewarson, Rosen, and Akyus and Utku (refs. 1 through 4). In 1969, Cuthill and McKee's (ref. 5) algorithm arranged the rows of the connectivity matrix with regard to the increasing number of nonzero off-diagonal elements. This algorithm was used in a program called BANDIT which serves a a preprocessor for NASTRAN.

H.R. Groom's algorithm for bandwidth reduction was introduced in 1972 (ref. 6). Groom systematically moved closer together rows and columns which were far apart and coupled.

In 1973, Collins (ref. 7) presented the algorithm upon which the program, BANSAP, developed in this study is based. After work on BANSAP had begun, Rodrigues (ref. 8) presented a new algorithm which, for two sample problems presented, showed a smaller bandwidth than the Cuthill and McKee, the Groom, or the Collins algorithms.

The objective of this paper is to describe a study undertaken to incorporate the Collins bandwidth algorithm in a data preprocessing computer program for the finite element program SAP IV (Structural Analysis Program - IV). First to be presented will be Collins' algorithm for bandwidth reduction which contains two subroutines, SETUP and OPTNUM. A description of the SAP IV preprocessing program BANSAP will then be given including its capabilities and limitations. Results from application of the

program to example problems will be presented and discussed. User instructions, the BANSAP program listing, and sample output are presented in appendices.

COLLINS BANDWIDTH REDUCTION ALGORITHM

Collins' algorithm for bandwidth reduction includes two subroutines, SETUP and OPTNUM. His procedure shall be illustrated using the structure in figure 1.

In the first subroutine, SETUP, a list is generated showing the connections between the different nodes shown in figure 1. The relations established by SETUP are displayed in table 1. The information is stored in arrays suitable for use in subroutine OPTNUM. The subroutine SETUP also determines the original bandwidth of the structure.

The subroutine used to renumber related nodes is OPTNUM. OPTNUM locates the origin of the different numbering schemes at each node in turn, making the number of permutations of schemes equal to the number of nodes. In other words, OPTNUM first renumbers the nodes around old node number one making old node number one the origin of the new scheme. OPTNUM then determines the bandwidth of this scheme. Next OPTNUM goes to old node number two and starts its new origin in the position of this node. It renumbers the nodes connecting node two, one at a time, and determines the maximum difference between the new connected nodes. If the maximum difference is less than the lowest maximum difference of the preceding schemes, it continues with the renumbering until the scheme is complete. If not, the current scheme is abandoned. After completion or abandonment of a scheme, OPTNUM proceeds to the next scheme starting with a new origin at the next sequential old node number. The scheme which is retained by OPTNUM is that which exhibits the lowest maximum difference between related nodes. The sequence of renumbering schemes for figure 1 is shown in table 2.

Collins' algorithm is set up to handle the renumbering of nodes for elements containing up to four nodes. Reference 5 indicates that this method has been applied to solid elements but not very successfully.

SAP IV PREPROCESSING PROGRAM

A program, BANSAP, has been written using the Collins algorithm as a preprocessing program for SAP IV. BANSAP consists of four subroutines: SAPIN, SETUP, OPTNUM, and SAPOUT as shown in figure 5.

The first subroutine, SAPIN, reads the data in the formats stipulated by SAP IV and stores element and node connections according to type. BANSAP is set up to handle two basic types of finite elements: elements connecting two nodes, and elements connecting three or four nodes. The two node elements which can be entered into subroutine SAPIN are either the truss, beam, or boundary. The actual renumbering of a two node element is the same for either element. The only difference in the handling of these elements by BANSAP is in their SAP IV formats. The three or four node elements which may be entered into subroutine SAPIN include membranes, axisymmetric two-dimensional elements and plate bending elements. Again, the only difference in the handling of these three and four node elements is in their SAP IV formats. If more than one type of element comprises the structure, the elements may be grouped according to their type. As is required for SAP IV, nodes must be sequentially numbered from one.

From subroutine SAPIN, BANSAP goes on to subroutines SETUP and OPTNUM. The new bandwidth is printed and a list of old number node numbers and new numbers is generated. As a user option the subroutine SAPOUT will punch the original elements with the new node numbers. Program BANSAP has been dimensioned in this paper to permit up to 1000 nodes and 1000 elements.

APPLICATIONS OF BANSAP

Applications of BANSAP are presented in table 3. The first two problems shown are illustrations of reduction in bandwidth which may be attained for simple problems. Problem 3 is an example taken from structural analysis of a ship radar tower. The last two entries are practical problems encountered in finite analysis of composite material structures.

The first illustration is the sample finite element scheme shown in figure 1. After renumbering by BANSAP the bandwidth was reduced from 9 to 4 and the final scheme is shown in figure 3.

The truss problem shown in figure 6a is a wagonwheel. After processing by BANSAP the bandwidth was reduced from 9 to 6. It has been found, however, that this value is not the optimum bandwidth. Collins has noted that the wagonwheel problem is a special case and the true optimum bandwidth occurs when the node number of the hub of the wheel is set equal to half the number of spokes plus one. The optimum bandwidth of the wagonwheel shown in figure 6 is actually 5.

The third structure is the ship's radar tower shown in figure 7. The original numbering scheme shown is nearly optimum with a bandwidth of 12 since the renumbering scheme only reduces the bandwidth to 9. For such structures there is no appreciable gain by using BANSAP as the structures could easily be numbered by hand to obtain a small bandwidth.

The shear panel of figure 8 is an example of a greatly enlarged bandwidth which can occur from the addition of new finite elements after the original structure has been numbered. With the addition of new elements for the shear panel a bandwidth of 406 was obtained, but after BANSAP, the bandwidth was reduced to 35.

The bolted joint specimen illustrated in figure 9 is a good example of how BANSAP can be used to obtain an optimum bandwidth when the numbering scheme is difficult to select by hand. The

nodes of the bolted joint specimen were originally numbered to permit easier data generation using a FORTRAN program. After the cards had been generated, BANSAP renumbered the nodes to reduce the bandwidth from 168 to 28.

CONCLUDING REMARKS

A FORTRAN program has been written for bandwidth reduction by nodal renumbering. The program is based upon the Collins algorithm and serves as a data preprocessor for the finite element program SAP IV. Applications of the preprocessing program to a number of simple and realistic problems have been presented.

Nodal renumbering for finite element analysis may be required for a variety of reasons. Renumbering may be needed if new elements were to be added onto a previously numbered structure or if a structure is difficult to optimally number by hand. It may also be needed if the element and nodal data were prepared by data generation programs. Such reasons clearly show a need and use for a program such as BANSAP.

BANSAP is an effective preprocessing program for SAP IV. The algorithm used greatly reduces the bandwidth for reduced computer time and storage during the finite element analysis.

APPENDIX A

USER INSTRUCTIONS

USER INSTRUCTIONS

CONTROL CARD (3I5)

Columns	1 - 5	Number of different groups of elements
	6 - 10	Total number of nodes
	11 - 15	The number zero for nopunched output and any number greater than zero for punched output

The following types of elements are permitted in the program.

Type 1 TRUSS

CONTROL CARD (3I5)

Columns	1 - 5	The number 1
	6 - 10	The number of elements in group 1

Element Data Cards (3I5, 2A10)

Columns	1 - 5	Element number
	6 - 10	Node number I
	11 - 15	Node number J

Type 2 BEAM

CONTROL CARD (3I5)

Columns	1 - 5	The number 2
	6 - 10	The number of elements in group 2

Element Data Cards (4I5, 5A10)

Columns	1 - 5	Element number
	6 - 10	Node number I
	11 - 15	Node number J
	16 - 20	Node number K; K is any nodal point which lies in the local 1 - 2 plane but not on the 1 axis (see ref. 9, page iv.2.2)

Type 3 MEMBRANE

CONTROL CARD (3I5)

Columns	1 - 5	The number 3
	6 - 10	The number of elements in group 3

Element Data Cards (5I5, 5A10)

Columns	1 - 5	Element number
	6 - 10	Node number I
	11 - 15	Node number J
	16 - 20	Node number K
	21 - 25	Node number L

Type 4 TWO D

CONTROL CARD (3I5)

Columns	1 - 5	The number 4
	6 - 10	The number of elements in group 4

Element Data Cards (5I5, 5A10)

Columns	1 - 5	Element number
	6 - 10	Node number I
	11 - 15	Node number J
	16 - 20	Node number K
	21 - 25	Node number L

Type 6 PLATE

CONTROL CARD (3I5)

Columns	1 - 5	The number 6
	6 - 10	Number of plate elements

Element Data Cards (5I5, 5A10)

Columns	1 - 5	Element number
	6 - 10	Node number I
	11 - 15	Node number J
	16 - 20	Node number K
	21 - 25	Node number L

Type 7 BOUNDARY (LINEAR SPRING)

CONTROL CARD (3I5)

Columns	1 - 5	The number 7
	6 - 10	The number of elements in group 7

Element Data Cards (2I5, 6A10)

Columns	1 - 5	Node N, at which the element is placed
	6 - 10	Node I

APPENDIX B

BANSAP SOURCE LISTING

```
              PROGRAM BANSAP(INPUT,OUTPUT,PUNCH,
             *                   TAPE5=INPUT,TAPE6=OUTPUT,TAPE7=PUNCH)
000003        DIMENSION NEWJT(1000),JOINT(1000)
000003        COMMON         LMENTS,JT(4000),MEMJT(8000),JMEM(1000),JNT(1000)
000003        COMMON/BAND/IDIFF,MINMAX
000003        COMMON/CONTR/NELG,ITYPE(5),NEL(5),NODES,IPUNCH
000003        COMMON/JUNK/A(1000,6)
000003        COMMON/UNIT/ IN,IT,IP
000003        IN = 5
000004        IT = 6
000005        IP = 7
        C
        C       JMEM(I)  = NUMBER OF NODES TO WHICH A SINGLE NODE IS CONNECTEC
        C       JT(I)    = WORKING ARRAY
        C      -MEMJT(I) = IDENTITIES OF NODES TO WHICH A NODE IS CONNECTED
        C
000006        WRITE(IT,12)
000012     12 FORMAT(1H1,9(/),
             1 36X,52HBBBBB     AAAAA    N     N    SSSSS   AAAAA    PPPPP/
             2 36X,53HB     B  A     A   NN    N   S       A     A   P     P/
             3 36X,53HB     B  A     A   N N   N   S       A     A   P     P/
             4 36X,52HBBBBB    AAAAAAA   N  N  N    SSSS   AAAAAAA   PPPPP/
             5 36X,52HB     B  A     A   N   N N        S  A     A   P    /
             6 36X,52HB     B  A     A   N    NN        S  A     A   P    /
             7 36X,52HBBBBB    A     A   N     N   SSSSS   A     A   P    ,)
000012        WRITE(IT,16)
000016     16 FORMAT(1H1,5X,19H I N P U T  D A T A ,//,)
000016        DO 10 I=1,1000
000020     10 JNT (I)= 0
000023        DO 20 I=1,4000
000024     20 JT(I) = 0
000027        DO 30 I=1,8000
000030     30 MEMJT(I) =0
        C
000033        CALL SAPIN
        C
        C       SUBROUTINES SETUP AND OPTNUM FROM/
        C      -BANDWIDTH REDUCTION BY AUTOMATIC RENUMBERING-, R.J. COLLINS,
        C       INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN ENGINEERING
        C       VOLUME 6, 1973, PP 345-356.
        C
000034        CALL SETJP
```

```
       C
000035          WRITE(IT,32)
000041          DO 40 I= 1,NODES
000043          NO = JMEM(I)
000045          L1 = 8*(I-1) + 1
000047          L2 = L1 +NO -1
000051          WRITE(IT,34)  I,NO,(MEMJT(L),L=L1,L2)
000070       40 CONTINUE
000073          MINMAX = IDIFF + 1
000075          WRITE(IT,36) MINMAX
       C
000102          CALL OPTNUM
       C
000103          MINMAX = MINMAX +1
000105          WRITE(IT,38) MINMAX
000112          WRITE(IT,42)
       C
000116          CALL SAPOUT
       C
000117       32 FORMAT(1H1,12X,4HNODE,3X,4HJMEM,16X,5HMEMJT,//)
000117       34 FORMAT(11X,2I5,10X,9I6)
000117       36 FORMAT(//,20X,20HORIGINAL BANDWIDTH =,I4 )
000117       38 FORMAT(//,20X,14HNEW BANDWIDTH=,I4)
000117       42 FORMAT(1H1,10X,33HOLD NODE NUMBER    NEW NODE NUMBER,/)
000117          STOP
000121          END
```

```
             SUBROUTINE SAPIN
000002       COMMON       LMENTS,JT(4000),MEMJT(8000),JMEM(1000),JNT(1000)
000002       COMMON/BAND/IDIFF,MINMAX
000002       COMMON/CONTR/   NELG,ITYPE(5),NEL(5),NODES,IPUNCH
000002       COMMON/JUNK/A(1000,6)
000002       COMMON/UNIT/ IN,IT,IP
      C
      C
      C        NELG      = NUMBER OF DIFFERENT GROUPS OF ELEMENTS(LESS THAN 5
      C        NODES     = TOTAL NUMBER OF NODES
      C        IPUNCH    = ZERO FOR NO PUNCHED OUTPUT, NUMBER GREATER THAN
      C       ZERO FOR PUNCHED OUTPUT
      C        N         = ELEMENT TYPE
      C        NE        = NUMBER OF ELEMENTS OF TYPE N
      C        LMENT     = TOTAL NUMBER OF ELEMENTS
      C
      C        ITYPE(I)  = TYPE OF ELEMENT
      C        NEL(I)    = NUMBER OF ELEMENTS IN A GROUP
      C
      C              ITYPE   ELEMENT   NUMBER OF NODES
      C                1      TRUSS          2
      C                2      BEAM           2
      C                3     MEMBRANE        3 OR 4
      C                4     TWO D           3 OR 4
      C                5     BRICK           8
      C                6     PLATE           4
      C                7     BOUNDRY         2
      C
      C        READ ELEMENT CARDS AND STORE CONNECTIONS.
      C
000002       READ(IN,12) NELG,NODES,IPUNCH
000014       WRITE(IT,14) NELG,NODES,IPUNCH
000026       DO 200 II= 1,NELG
000030       READ(IN,12) N,NE
000037       WRITE(IT,10) II,NE,N
000051       WRITE(IT,50)
000055       ITYPE(II) =N
000057       NEL(II)   =NE
000061       LMENTS =0
      C
      C        READ ELEMENT CONNECTIONS.  FOR TRUSS, BEAM, OR BOUNOARY
      C       ELEMENTS ONLY TWO CONNECTIONS I AND J ARE NEEDED.  FOR
```

```
       C       ALL OTHER TYPES FOUR CONNECTIONS ARE POSSIBLE- I,J,K,L.
       C
       C         STORE NODE CONNECTIONS ACCORDING TO TYPE.
       C
000062           DO 210 JJ = 1,NE
000063           GO TO (1,2,3,3,5,3,7),N
       C
000075         1 CONTINUE
000076           READ(IN,102) I,J,( A(JJ,L),L=1,2)
000115       102 FORMAT(5X,2I5,2A10)
000116           GO TO 300
       C
000117         2 CONTINUE
000117           READ(IN,104) I,J,( A(JJ,L),L=1,6)
000137       104 FORMAT(5X,2I5,A5,5A10)
000137           GO TO 300
       C
000140         3 CONTINUE
000140           READ(IN,106) I,J,K,L,( A(JJ,L),L=1,5)
000164       106 FORMAT(5X,4I5,5A10)
000164           GO TO 300
       C
000165         5 CONTINUE
000165           WRITE(IT,108)
000171       108 FORMAT(5X,42HTHREE DIMENSIONAL ELEMENTS NOT IMPLEMENTED,/)
000171           GO TO 200
       C
000172         7 CONTINUE
000172           READ(IN,110) I,J,(A(JJ,L),L=1,6)
000212       110 FORMAT(2I5,6A10)
000212       300 CONTINUE
000212           IF(N.EQ.7) GO TO 20
000214           III = JJ  + LMENTS
000216           JJJ = III + 1000
000220           JT(III) = I
000221           JT(JJJ) = J
000223           IF( N.LE. 2) GO TO 205
000226           KKK = III + 2000
000230           LLL = III + 3000
000232           JT(KKK) = K
       C
       C         FOR TRIANGULAR ELEMENT SET REPEATED NODE NUMBER EQUAL TO ZERO
       C
```

```
000234          IF(K.EQ.L)  L=0
000236          JT(LLL) = L
000240      205 CONTINUE
000240          WRITE(IT,30) JJ,JT(III),JT(JJJ),JT(KKK),JT(LLL)
000256      210 CONTINUE
000261          LMENTS = LMENTS + NE
000262      200 CONTINUE
000265       50 FORMAT(10X,7HELEMENT,5X,1HI,7X,1HJ,7X,1HK,7X,1HL,//)
000265       10 FORMAT(/,5X,13HELEMENT GROUP,I2,4H HAS,I3,17H ELEMENTS OF TYPE,
               1/)
000265       12 FORMAT(3I5)
000265       14 FORMAT(    ,2(/)
               1 10X,25HNUMBER OF ELEMENT TYPES =,I5,/
               2 10X,25HNUMBER OF NODAL POINTS  =,I5, /
               3 10X,25HPUNCHED ELEMENT CARDS   =,I5,/
               4 10X,25H            .EQ. 0  NO    ,/
               5 10X,25H            .EQ. 1  YES   ,)
000265       30 FORMAT(10X,I5,4I8)
000265          RETURN
000266          END
```

```
                SUBROUTINE SETUP
000002          COMMON       LMENTS,JT(4000),MEMJT(8000),JMEM(1000),JNT(1000)
000002          COMMON/BAND/IDIFF,MINMAX
000002          COMMON/CONTR/NELG,ITYPE(5),NEL(5),NODES,IPUNCH
         C
         C        NODES     = TOTAL NUMBER OF NODES
         C        JNTI      = ELEMENT NODE UNDER CONSIDERATION
         C        JSUB      = LOCATION IN MEMJT(I) OF BEGINNING OF LIST OF
         C       NODES RELATED TO JNTI
         C        LMENT     = TOTAL NUMBER OF ELEMENTS
         C        JMEM(I)   = NUMBER OF NODES TO WHICH A SINGLE NODE IS CONNECTEI
         C        MEMJT(I)  = IDENTITIES OF NODES TO WHICH A NODE IS CONNECTED
         C        IDIFF     = BANDWIDTH = IDIFF+1 FOR ORIGINAL SCHEME
         C
000002          IDIFF = 0
000003          DO 10 J= 1,NODES
000005       10 JMEM(J)= 0
000011          DO 60 J= 1,LMENTS
000012          DO 50 I= 1,4
         C
         C        NEXT STATEMENT DEPENDS ON THE NUMBER OF NODES FOR WHICH THE
         C       PROGRAM IS DIMENSIONED.  CURRENTLY THE MAXIMUM NUMBER OF NODES
         C        IS 1000.
         C
000013          JNTI = JT(1000* (I-1) + J)
         C
         C        IF JNTI EQUALS ZERO ALL NODES OF ELEMENT J HAVE BEEN
         C       CONSIDERED.
         C
000017          IF(JNTI.EQ.0) GO TO 60
000021          JSUB = (JNTI - 1) * 8
000023          DO 40 II = 1,4
000025          IF(II.EQ.I) GO TO 40
         C
         C        NEXT STATEMENT DEPENDS ON THE NUMBER OF NODES FOR WHICH THE
         C       PROGRAM IS DIMENSIONED.  CURRENTLY THE MAXIMUM NUMBER OF NODES
         C        IS 1000.
         C
         C        RELATED NODES ARE IDENTIFIED BELOW.
         C
000027          JJT =JT(1000 * (II-1) +J )
000033          IF(JJT.EQ.0) GO TO 50
```

```
C
C        DETERMINE WHETHER RELATIONSHIP BETWEEN JNTI AND JJT
C       HAS BEEN ESTABLISHED.
C
         MEM1 = JMEM(JNTI)
         IF(MEM1.EQ.0) GO TO 30
         DO 20 III =1,MEM1
         IF(MEMJT(JSUB +III).EQ.JJT) GO TO 40
C
C        FIND WIDTH OF ORIGINAL MATRIX BAND.
C
      20 CONTINUE
      30 JMEM(JNTI) =JMEM(JNTI) + 1
         IDUM = JSUB + JMEM(JNTI )
         MEMJT(IDUM) = JJT
         IF(IABS(JNTI-JJT).GT.IDIFF) IDIFF = IABS(JNTI -JJT)
      40 CONTINUE
      50 CONTINUE
      60 CONTINUE
         RETURN
         END
```

```
              SUBROUTINE OPTNUM
000002        DIMENSION NEWJT(1000),JOINT(1000)
000002        COMMON       LMENTS,JT(4000),MEMJT(8000),JMEM(1000),JNT(1000)
000002        COMMON/BAND/IDIFF,MINMAX
000002        COMMON/CONTR/NELG,ITYPE(5),NEL(5),NODES,IPUNCH
000002        COMMON/UNIT/ IN,IT,IP
        C
        C       JOINT(I) = WORKING ARRAY
        C       NEWJT(I) = WORKING ARRAY
        C       JNT(I)   = NEW NUMBERING SCHEME
        C      -MINMAX   = BANDWIDTH = MINMAX+1 FOR NEW SCHEME
        C
        C       MINMAX IS INITIALIZED.
        C
000002        MINMAX = IDIFF
        C
        C       NEW SCHEME STARTS AT NODE OF OLD NODE NUMBER IK.
        C
000004        DO 60 IK=1,NODES
000005        DO 20 J =1,NODES
        C
        C       JOINT(J) AND NEWJT (J) INITIALIZED TO ZERO FOR EACH NEW
        C      NUMBERING SCHEME.
        C
000006        JOINT(J)= 0
000007     20 NEWJT(J)= 0
        C
        C       INITIALIZE FOR NEW NODE NUMBER ONE.
        C
000013        MAX =0
000013        I= 1
000014        NEWJT(1) =IK
000016        JOINT(IK)= 1
000017        K = 1
000021     30 CONTINUE
000021        JDUM = NEWJT(I)
000023        K4 = JMEM(JDUM)
000025        IF(K4.EQ.0) GO TO 45
        C
        C       LOCATE RELATED NODES IN MEMJT(I).
        C
000026        JSUB = (NEWJT(I) -1) *8
```

```
000031          DO 40 JJ= 1,K4
000032          K5 = MEMJT(JSUB +JJ)
000034          IF( JOINT(K5) .GT. 0 ) GO TO 40
000040          K = K+1
000041          NEWJT(K) =K5
000042          JOINT(K5)=K
       C
       C          CHECK DIFFERENCE BETWEEN NEW NUMBERS OF RELATED NODES.
       C
000043          NDIFF = IABS(I-K)
       C
       C          SCHEME ABANDONED IF DIFFERENCE GREATER THAN BANDWIDTH OF
       C         PRIVIOUS SCHEME, NEW SCHEME STARTED.
       C
000045          IF(NDIFF.GE.MINMAX) GO TO 60
000050          IF(NDIFF.GT.MAX) MAX =NDIFF
000053       40 CONTINUE
000056          IF(K.EQ.NODES)GO TO 50
000060       45 I=I+1
000062          GO TO 30
000062       50 MINMAX = MAX
000064          DO 55 J=1,NODES
000065       55 JNT(J) = JOINT(J)
000071       60 CONTINUE
000074          RETURN
000075          END
```

```
              SUBROUTINE SAPOUT
000002        COMMON        LMENTS,JT(4000),MEMJT(8000),JMEM(1000),JNT(1000)
000002        COMMON/CONTR/ NELG,ITYPE(5),NEL(5),NODES,IPUNCH
000002        COMMON/JUNK/A(1000,6)
000002        COMMON/UNIT/IN,IT,IP
000002        DO 10 I= 1,NODES
000004        WRITE(IT,12) I,JNT(I)
000013     10 CONTINUE
000016        LMENTS = 0
000017        WRITE(IT,14)
000022        DO 20 II=1,NELG
000024        N = ITYPE(II)
000026        NE= NEL(II)
000027        DO 21 JJ =1,NE
000031        I = JT( JJ+LMENTS      )
000034        J = JT( JJ+LMENTS +1000)
000036        NI=  JNT(I)
000040        NJ=  JNT(J)
       C
       C        OUTPUT NODE CONNECTIONS ACCORDING TO TYPE.
       C
000042        GO TO (1,2,3,3,5,3,7),N
       C
000055      1 CONTINUE
000055        WRITE(IT,102) JJ,NI,NJ,(A(JJ,L),L=1,2)
000077        IF(IPUNCH.GT.0)
             *WRITE(IP,102) JJ,NI,NJ,(A(JJ,L),L=1,2)
000122    102 FORMAT(3I5,2A10)
000122        GO TO 21
       C
000123      2 CONTINUE
000123        K = A(JJ,1)
000126        NK= JNT(K)
000130        WRITE(IT,104) JJ,NI,NJ,NK,(A(JJ,L),L=2,6)
000153        IF(IPUNCH.GT.0)
             *WRITE(IP,104) JJ,NI,NJ,NK,(A(JJ,L),L=2,6)
000200    104 FORMAT(4I5,5A10)
000200        GO TO 21
       C
000201      3 CONTINUE
000201        K = JT( JJ+LMENTS +2000)
000204        L = JT( JJ+LMENTS +3000)
```

```
000206           IF(L.EQ.0)  L=K
000210           NK= JNT(K)
000212           NL= JNT(L)
000214           WRITE(IT,106)JJ,NI,NJ,NK,NL,(A(JJ,L),L=1,5)
000242       106 FORMAT(5I5,5A10)
000242           IF(IPUNCH.GT.0)
                *WRITE(IP,106)JJ,NI,NJ,NK,NL,(A(JJ,L),L=1,5)
000271           GO TO 21
           C
000272         5 CONTINUE
000272           GO TO 21
           C
000273         7 CONTINUE
000273           WRITE(IT,108)NI,NJ,(A(JJ,L),L=1,6)
000313           IF(IPUNCH.GT.0)
                *WRITE(IP,108)NI,NJ,(A(JJ,L),L=1,6)
000334       108 FORMAT(2I5,6A10)
000334        21 CONTINUE
000337           LMENTS = LMENTS +NE
000340        20 CONTINUE
000342        12 FORMAT(15X,I5,13X,I5)
000342        14 FORMAT(1H1,//,4X,17HNEW ELEMENT CARDS,//)
000342        30 FORMAT( 5I5)
000342           RETURN
000343           END
```

APPENDIX C

SAMPLE BANSAP OUTPUT

```
BBBBB     AAAAA    N     N    SSSSS    AAAAA    PPPPP
B    B   A     A   NN    N   S        A     A   P    P
B    B   A     A   N N   N   S        A     A   P    P
BBBBB    AAAAAAA   N  N  N    SSSS    AAAAAAA   PPPPP
B    B   A     A   N   N N        S   A     A   P
B    B   A     A   N    NN        S   A     A   P
BBBBB    A     A   N     N   SSSSS    A     A   P
```

I N P U T   D A T A

```
NUMBER OF ELEMENT TYPES =      1
NUMBER OF NODAL POINTS  =      9
PUNCHED ELEMENT CARDS   =      0
           .EQ. 0  NO
           .EQ. 1  YES
```

ELEMENT GROUP 1 HAS 8 ELEMENTS OF TYPE 3

ELEMENT	I	J	K	L
1	1	9	8	-0
2	1	2	9	-0
3	2	4	9	-0
4	2	3	4	-0
5	8	6	7	-0
6	8	9	6	-0
7	9	5	6	-0
8	9	4	5	-0

NODE	JMEM	MEMJT					
1	3	9	8	2			
2	4	1	9	4	3		
3	2	2	4				
4	4	2	9	3	5		
5	3	9	6	4			
6	4	8	7	9	5		
7	2	8	6				
8	4	1	9	6	7		
9	6	1	8	2	4	6	5

ORIGINAL BANDWIDTH = 9

NEW BANDWIDTH= 4

OLD NODE NUMBER	NEW NODE NUMBER
1	4
2	2
3	1
4	3
5	6
6	8
7	9
8	7
9	5

NEW ELEMENT CARDS

1	4	5	7	7
2	4	2	5	5
3	2	3	5	5
4	2	1	3	3
5	7	8	9	9
6	7	5	8	8
7	5	6	8	8
8	5	3	6	6

REFERENCES

1. "An Algorithm for Reducing the Bandwidth of a Matrix of Symmetrical Configuration", by Always, G.G. and Martin, D.W., Computer Journal, Volume 8, 1965, pp. 264-272.

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8. "Node Numbering Optimization in Structural Analysis", by Rodrigues, J.S., Journal of the Structural Division, ASCE, Volume 101, 1975, pp. 361-375.

9. "SAP IV, A Structural Analysis Program for Static and Dynamic Response of Linear Systems", by Bathe, K., Wilson, E.L., and Peterson, F.E., Report No. EERC 73-11, June 1973.

Table 1. Example node connections determined in subroutine SETUP.

Node	Number of Connected Nodes	Connected Nodes
1	3	9, 8, 2
2	4	1, 9, 4, 3
3	2	2, 4
4	4	2, 9, 3, 5
5	3	9, 6, 4
6	4	8, 7, 9, 5
7	2	8, 6
8	4	1, 9, 6, 7
9	6	1, 8, 2, 4, 6, 5

Table 2. Trail Numbering Schemes Used in OPTNUM.

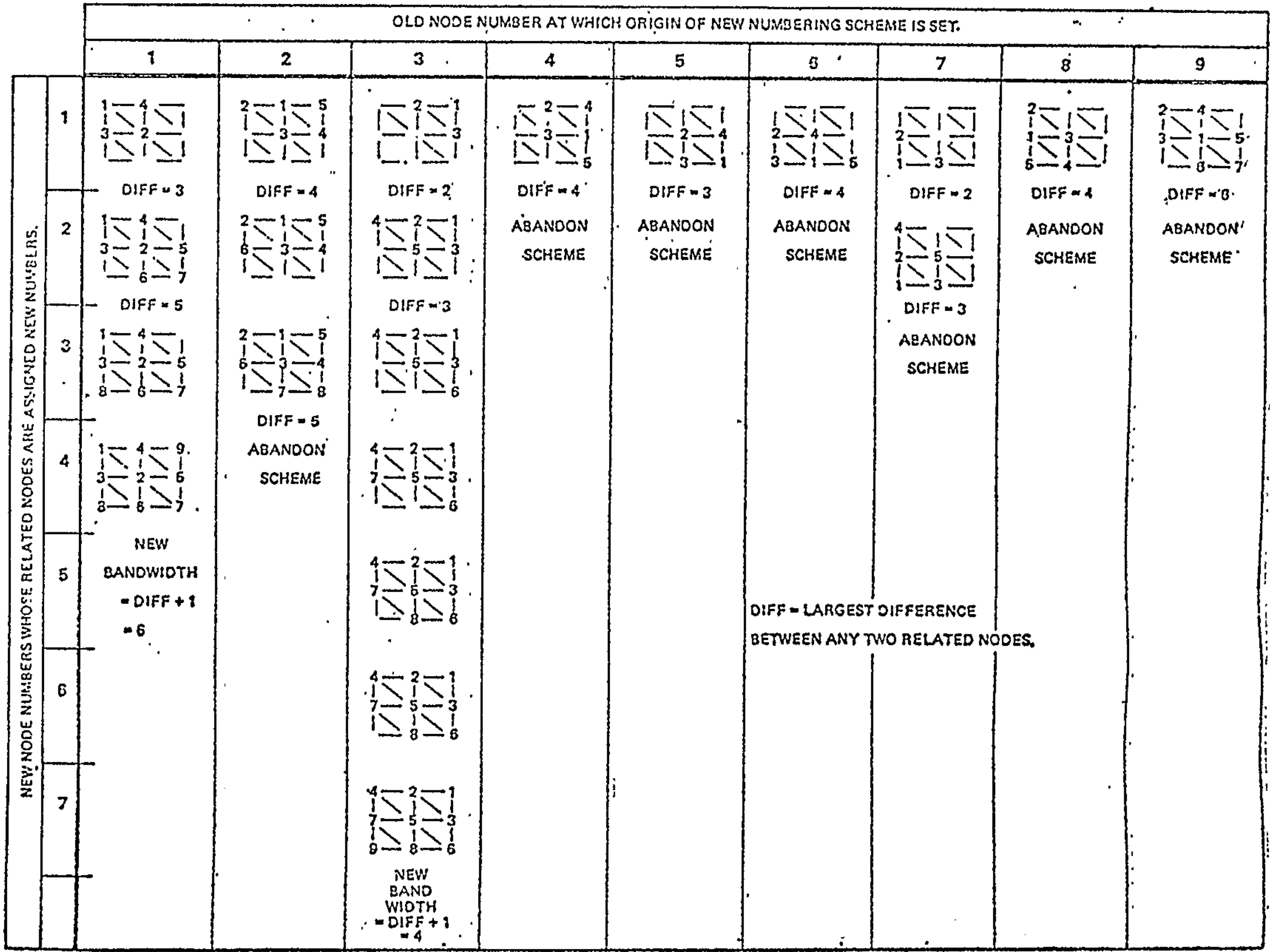


Table 3. Summary of applications of BANSAP.

Structure	Element Type	Number of Nodes	Number of Elements	Old Band-width	New Band-width
Sample problem Figure 1	Membrane	9	8	9	4
Wagonwheel Figure 6	Truss	9	16	9	6
Ship tower Figure 7	Beam	25	65	12	9
Shear panel specimen Figure 8	Membrane	595	554	406	35
Bolted joint specimen Figure 9	Membrane	398	349	168	28

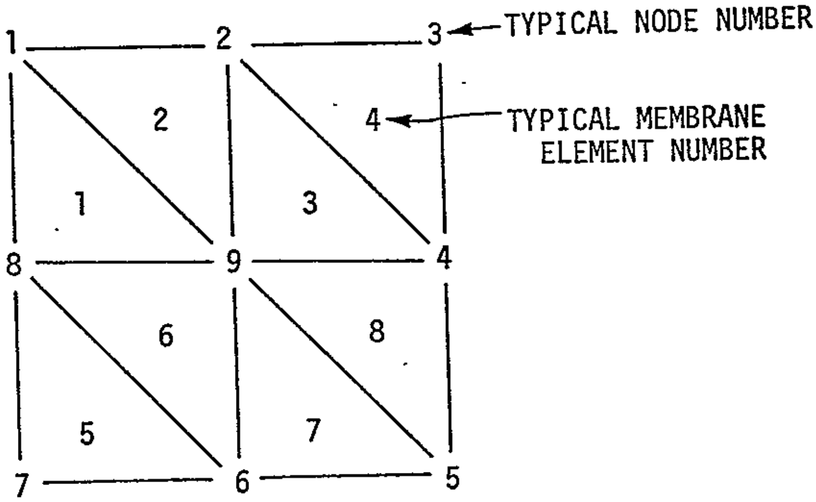


Figure 1. Sample Finite Element Scheme

NODE

NODE	1	2	3	4	5	6	7	8	9
1	X	X						X	X
2	X	X	X	X					X
3		X	X	X					
4		X	X	X	X				X
5				X	X	X			X
6					X	X	X	X	X
7						X	X	X	
8	X					X	X	X	X
9	X	X		X	X	X		X	X

Figure 2. Connectivity Matrix of Sample Scheme.

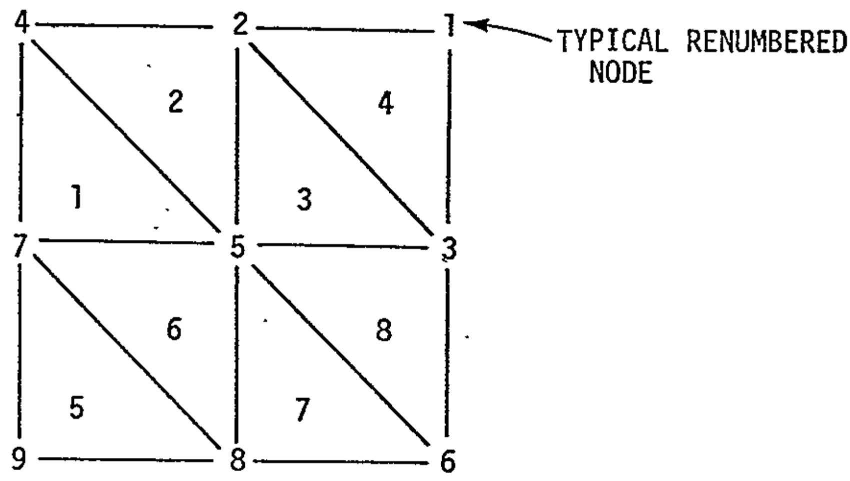


Figure 3. Renumbered Finite Element Scheme.

NODE

NODE	1	2	3	4	5	6	7	8	9
1	X	X	X						
2	X	X	X	X	X				
3	X	X	X		X	X			
4		X		X	X		X		
5		X	X	X	X	X	X	X	
6			X		X	X		X	
7				X	X		X	X	X
8					X	X	X	X	X
9							X	X	X

Figure 4. Connectivity Matrix of Renumbered Scheme.

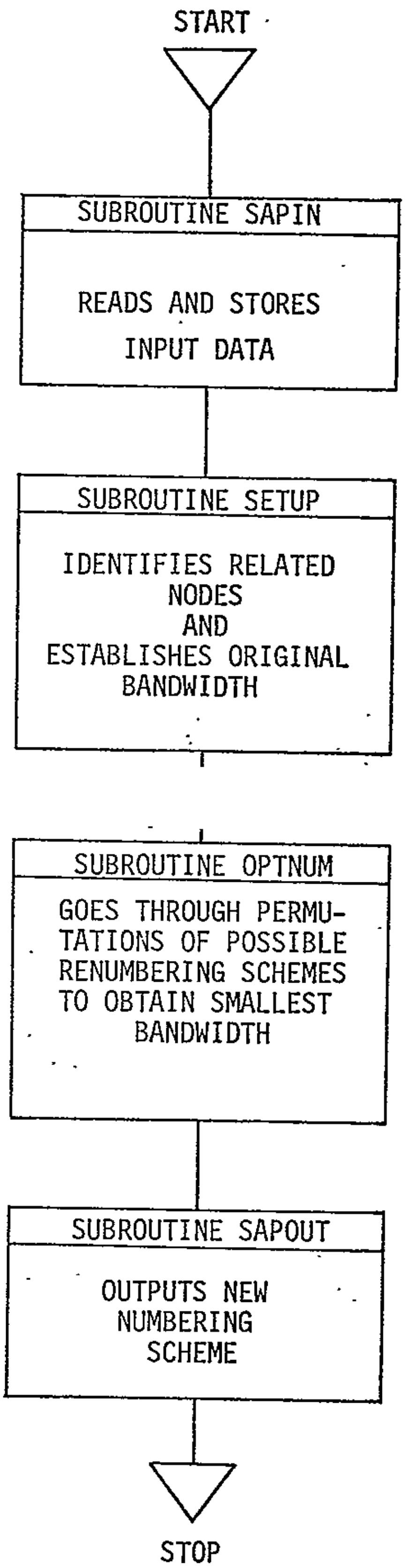
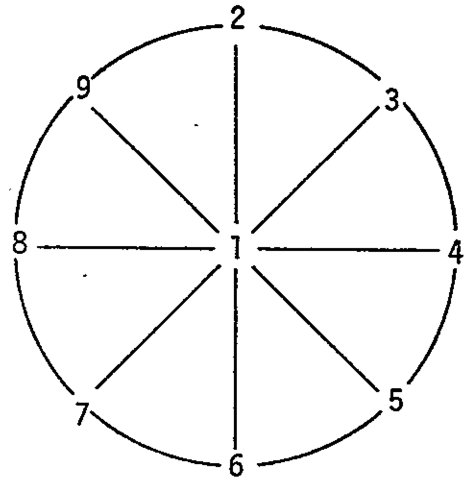
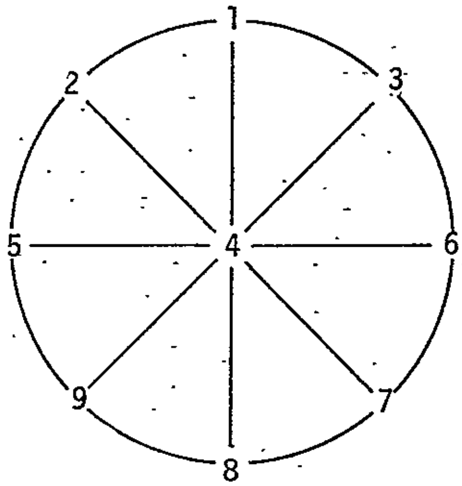


Figure 5. Flowchart of SAP IV BANSAP Preprocessing Program.



(a) Original Scheme



(b) Renumbered Scheme

Figure 6. Wagonwheel Truss.

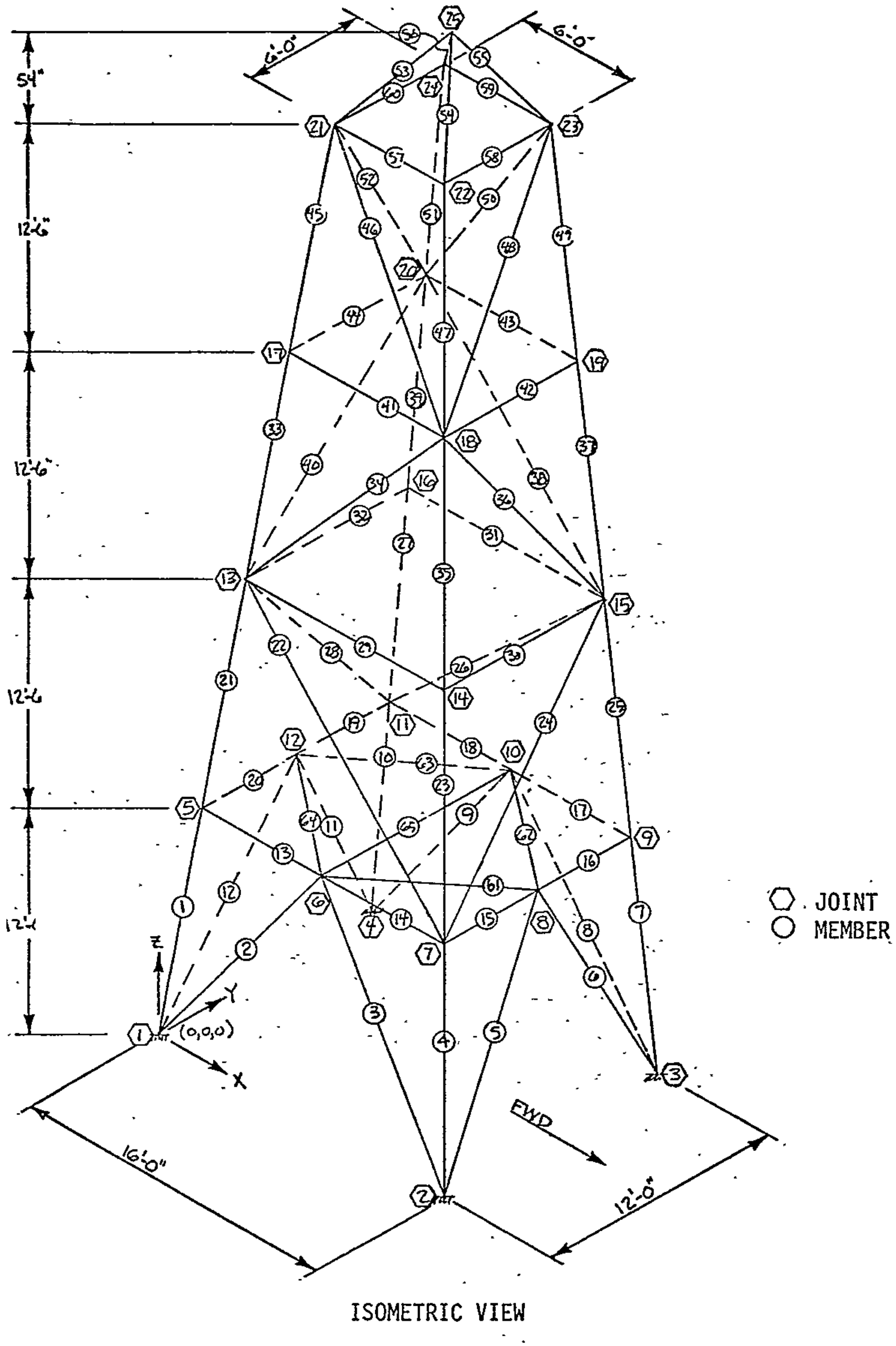


Figure 7. Ship Radar Tower.

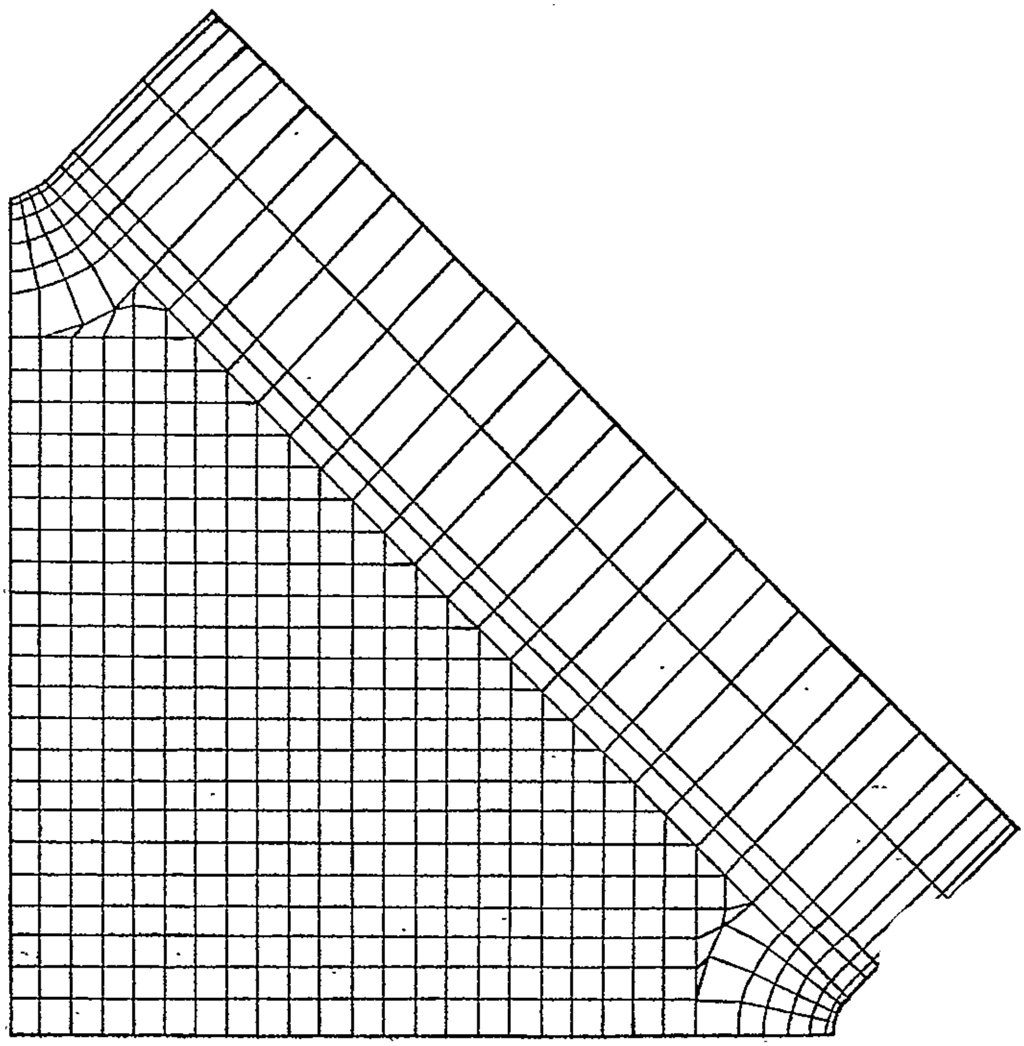


Figure 8. Shear Panel Specimen.

TRIANGULAR AND QUADRILATERAL MEMBRANE ELEMENTS

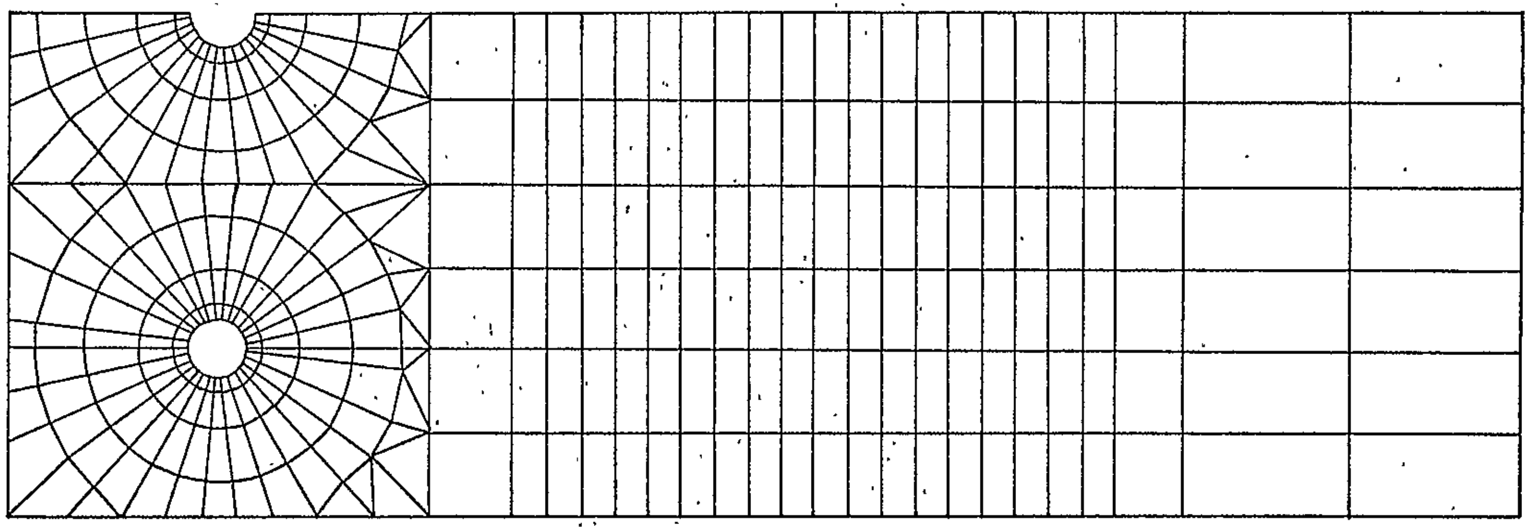


Figure 9: Finite Element Mesh for Composite Bolted Joint Specimen.

FEMESH: A FINITE ELEMENT MESH GENERATION PROGRAM BASED ON ISOPARAMETRIC ZONES

By

Zoa C. Lane

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INTRODUCTION

Finite element analysis programs greatly facilitate the determination of deformations and stresses in structures. A major inconvenience in utilizing this analysis technique is the large amount of input data required by the computer programs. This data includes, in addition to material characteristics, the node numbers defining the elements and the spatial coordinates for each node.

Current mesh generation methods include for simple problems data preparation by hand, and for more complex problems, the coding and executing of FORTRAN mesh generation programs which generate data for a general structure.

W.R. Buell and W.A. Bush surveyed some techniques used in current mesh generation schemes (ref. 1). The techniques presented by Buell and Bush are: a straight line interpolation technique, a sides and parts technique for axisymmetric structures electro-mechanical techniques for two- and three-dimensional structures, and a simplified finite difference technique and equipotential technique for general structure shapes.

The advantages of general structure mesh generation programs (ref. 1) are: (1) reduced cost due to reduction of man hours and computer time needed to generate and check data; (2) reduced number of errors; (3) insured regularity of finite elements; and (4) application to a variety of structural shapes.

O.C. Zienkiewicz (ref. 2) utilizes a technique involving the mapping of isoparametric quadrilaterals from a natural to a cartesian coordinate system in an automatic mesh generation

scheme for plane and curved surfaces. This scheme is applicable to non-quadrilateral structures if the structure is divided into quadrilateral regions. Zienkiewicz's technique for mesh generation was used by S.J. Womack (ref. 3) as a preprocessor for TEXGAP, a finite element program for the analysis of two-dimensional linearly elastic plane or axisymmetric bodies (ref. 4).

The objective of this study is to utilize the technique developed by Zienkiewicz in a mesh generation scheme for two-dimensional planar surfaces. Presented in this paper are a description of the mapping technique, a description of the computer program, and three examples of meshes generated by the program. A set of user instructions and a listing of the program are included in the appendices.

INTERPOLATION FUNCTION TECHNIQUE FOR FINITE ELEMENT GENERATION

The algorithm used by Zienkiewicz to map an isoparametric quadrilateral is the displacement interpolation equations used in isoparametric finite elements (ref. 5). The interpolation equations for quadratic bounded surfaces (which are listed in table 1), are a function of a set of dimensionless coordinates, ξ and η, which define a natural coordinate system.

In the natural coordinate system (fig. 1), a planar surface is represented as a square whose dimensions are 2 x 2 units and whose center is at the origin. To map a surface into the cartesian coordinate system, eight boundary points (x_i and y_i) and the ξ and η values of each grid point on the surface to be mapped are substituted in the displacement interpolation functions; the resulting values are the cartesian coordinates of the grid points.

A mesh is generated by dividing the square into the desired number of subdivisions, calculating the ξ and η coordinates for each grid point, and mapping each point to the cartesian coordinate system. A graduation of a generated mesh is obtained

by offsetting the midside node from the midpoint of a side of the quadrilateral (fig. 2). The generated elements will vary in size along that side; smaller elements will be in the direction of the offset.

Meshes for complex structures are generated by dividing the structure into quadrilateral zones. The mesh for each zone is generated independent of other zones. Connection of zones is accomplished by eliminating node numbers and coordinates which were duplicated on zone boundaries.

PROGRAM FEMESH

Program FEMESH is a FORTRAN IV code for generating finite element data for two-dimensional planar surfaces. The algorithm used to generate the node coordinates is based on the displacement interpolation functions (table 1) described in the preceding paragraph.

Input data for FEMESH includes a title, the number of zones, the total number of zone nodes, the number of zone node coordinates to be read from cards, the first node and element numbers, a list of the eight nodes which define a zone, the dimensions of the desired mesh of each zone, and the zone node coordinates.

A zone is a quadrilateral region whose geometry is defined by eight zone nodes. (Zone nodes are used only in the input definition of the geometry; they are not included in the generated mesh.) The zone nodes are listed in counter-clockwise order. As indicated in figure 3, the first node identifies a corner of the quadrilateral. The second, fourth, sixth, and eighth nodes are referred to as midside nodes. If a midside node does not lie on the midpoint of a side, a graduation of the mesh results.

The general flow for the mesh generation program, FEMESH, is shown in figure 4. As indicated, the mesh for each zone is generated separately. The first step in the mesh generation scheme is to determine if the coordinates of the midside

nodes are defined (i.e., if their coordinates were supplied by the user). If the coordinates are not defined, the midside node is assumed to lie at the midpoint of a linear line segment. The second step is to determine if either of the four sides of the zone is connected to a zone for which a mesh was previously generated. If a side is connected to such a zone the node numbers and the x and y coordinates which have already been generated are used. The remainder of the mesh is then generated. This process is repeated until the meshes for all zones are generated.

The output of program FEMESH includes a listing of the elements, their four node numbers and the node coordinates. A plot of the mesh is also generated.

APPLICATIONS

Three finite element mesh generated by FEMESH are presented in this section. The first example is a sample problem illustrating the input and output of program FEMESH. The second is a quarter section of a shear panel. The third is a half section of a bolted joint specimen.

The first example is a simple structure originally used to validate the ability of FEMESH to properly connect zones. The structure (illustrated in fig. 5a) is divided into three zones. The eighteen zone nodes are labeled arbitrarily and illustrated in figure 5b. Figures 5a and 5b represent the input required by program FEMESH to generate the mesh illustrated in figures 5c and 5d. Figure 5c illustrates the node numbers, and 5d illustrates the element numbers.

The input data for this problem is tabulated in table 2 (see Appendix A for user instructions). The data includes a title card, a control card, three zone description cards, and eight node coordinate cards. The control card specifies the number zones (3), the number of zone nodes (18), the number of zone node coordinate cards to be read (8), the first node

number (100), and the first element number (1000). A typical zone description card lists the eight zone nodes defining each zone and the size of the finite element mesh to be generated.

The tabulated output for this problem appears in table 3. The output includes the input data, the element number, the four node numbers which define each element, and the cartesian coordinates of each node.

The shear panel illustrated in figure 6 is divided into four zones. The zones were established in such a way that the straight and curved segments of the corner fillets are assigned to different zones in order to obtain a closer approximation of the true boundary shape.

The mesh dimension for zone I is 20 x 20, for zone II is 20 x 3, for zone III is 20 x 20, and zone IV is 3 x 30. To avoid the generation of long, narrow rectangular elements, the midside nodes 2, 8, 9, and 14, 15, 16 are moved away from the midpoint of the line segment toward the fillets. The input data is summarized in table 4. The output is illustrated in figure 7. Because of the large number of generated elements, the output is not listed in tabular form; it is represented graphically by a computer plot of the generated mesh. The generated mesh is composed of 574 nodes and 520 elements.

The mesh for one-half of a bolted joint specimen was generated by dividing the specimen into 15 zones as illustrated in figure 8. The input data for this problem (table 5) consisted of 58 data cards, including 15 zone description cards, and 41 node coordinate cards. A graduation of the mesh of zones II, III, IV, V, VI, VIII, and IX was used to obtain a uniformity in the shape of the generated elements. The generated mesh, which is illustrated in figure 9, consists of 378 elements and 435 nodes.

CONCLUDING REMARKS

Program FEMESH, a FORTRAN IV code, has been developed to generate a finite element mesh for two-dimensional, planar

surfaces. The algorithm used is the displacement interpolation functions which were developed for mesh generation by Zienkiewicz.

A structure may be subdivided into a maximum of 15 zones. The maximum mesh for each zone is 24 x 24 elements (or 25 x 25 node points). FEMESH will compute a maximum of 4000 node points, and output the node numbers and their coordinates and the element numbers and their four identifying node numbers. A simple plot of the finite element mesh is also generated.

Presented in this paper is a description of the technique used in the mesh generation scheme, a description of program FEMESH and examples of the mesh generated for three problems. User instructions and a listing of the program are included in the appendices.

APPENDIX A

USER INSTRUCTIONS FOR FEMESH

USER INSTRUCTIONS FOR FEMESH

Program FEMESH generates isoparametric finite element meshes for two-dimensional planar surfaces. The input required by the program consists of four types of data cards: a title card, a control card, zone description cards, and node coordinate cards (fig. A1).

TITLE CARD (Format 10A4).

Column	Variable	Description
1-40	TITLE	Heading for output

CONTROL CARD (Format 6I5):

Column	Variable	Description
1-5	IZ	Number of zones (IZ $\leq$ 15)
5-10	NT	Total number of zone nodes
11-15	NI	Number of zone node coordinates to be read as input on cards
16-20	INODE	First node number to be assigned to generated mesh
21-25	IELM	First element number to be assigned to generated mesh
26-30	IP	Punch indicator: 0 will not punch 1 punch

A zone is a quadrilateral with either linear or curved line segments. The geometry of the zone is defined by 8 zone nodes whose coordinates are supplied by the user (see node coordinate card).

The values of NI and NT may differ due to the ability of the program to linearly interpolate to define the coordinates of the midside node if those coordinates are not supplied by the user. Midside nodes are those zone nodes which lie between two corner nodes. It is not necessary that a midside node lie at the midpoint of a line segment.

ZONE DESCRIPTION CARD (Format 10I5):

Column	Variable	Description
1-5	NODE (I,1)	Zone nodes defining zone geometry
6-10	NODE (I,2)	
11-15	NODE (I,3)	I is the zone number
16-20	NODE (I,4)	
21-25	NODE (I,5)	
26-30	NODE (I,6)	
31-35	NODE (I,7)	
36-40	NODE (I,8)	
41-45	M	Number of subdivisions along the side defined by 1st, 2nd, and 3rd zone nodes
46-50	N	Number of subdivisions along the side defined by 3rd, 4th, and 5th zone nodes.

Zone numbers are determined by the order of the zone description cards. The first zone description card is assigned the number one, the second is assigned the number two, etc.

The interconnectivity of zones is indicated by assigning a negative magnitude to zone nodes which lie on a side connected to a zone with a smaller zone number. For example, if 4 zones are connected as shown in figure A2, then the first eight values of the zone description cards should be:

Card 1:	1	2	3	7	11	10	9	6
Card 2:	-3	4	5	8	13	12	-11	-7
Card 3:	-11	-12	-13	16	21	20	19	15
Card 4:	-9	-10	-11	-15	-19	18	17	14

A side which is divided into M subdivisions must not be connected to a side divided into N subdivisions unless the values M and N are equal.

NODE COORDINATE CARD (Format I5, 2F10.5):

Column	Variable
1-5	Node number
5-15	x coordinate
16-25	y coordinate

This card may be omitted for any midside node which lies on a straight line if a graduation of the mesh is not desired.

A graduation in the mesh occurs when the midside node is offset from the midpoint of the line segment. The smaller elements will be in the same direction as the offset.

Due to a restriction in the FORTRAN coding, a midside node should not be assigned the coordinates (0,0) if the line segment is not a straight line.

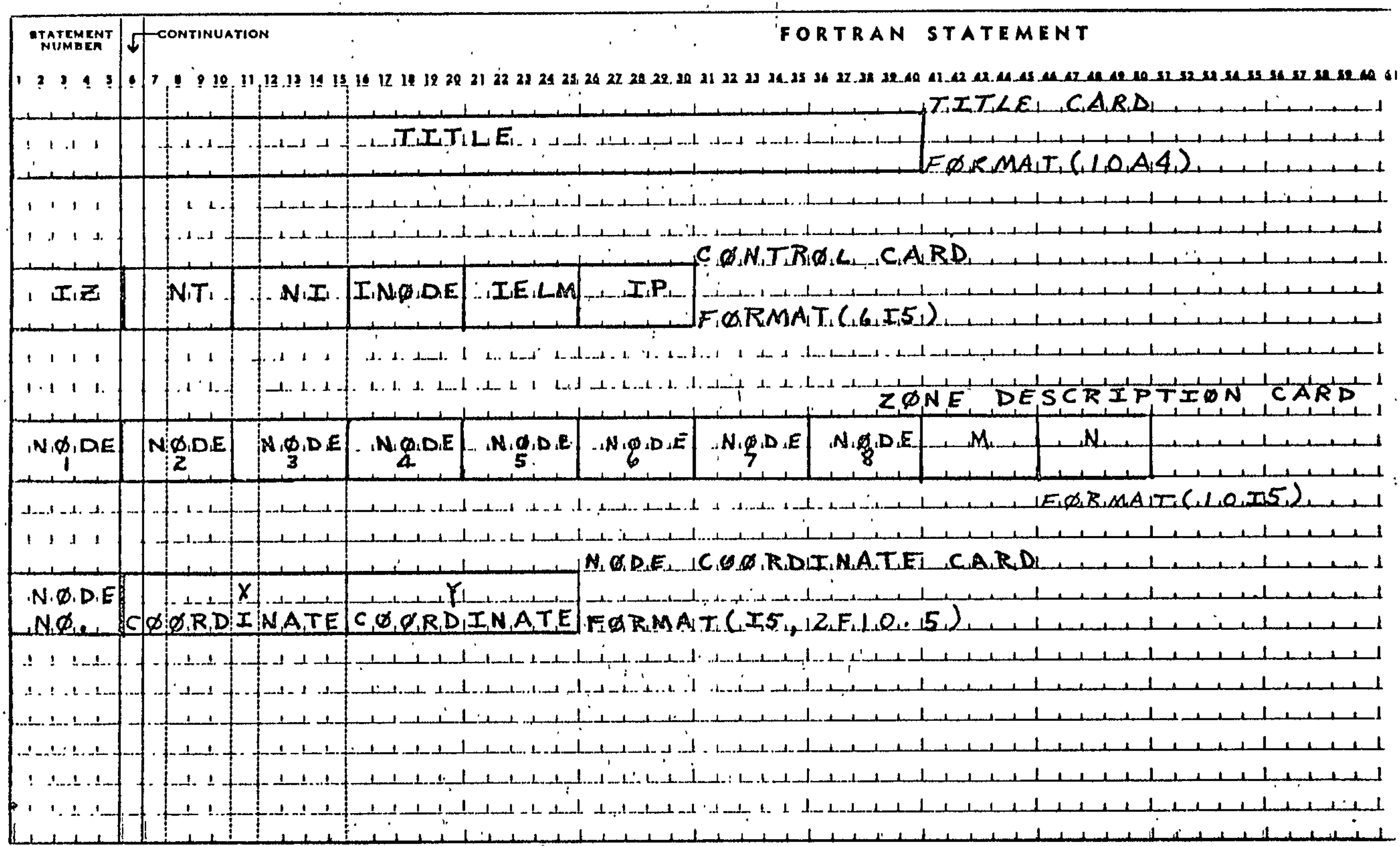


Figure A1. Input Data Formats for FEMESH.

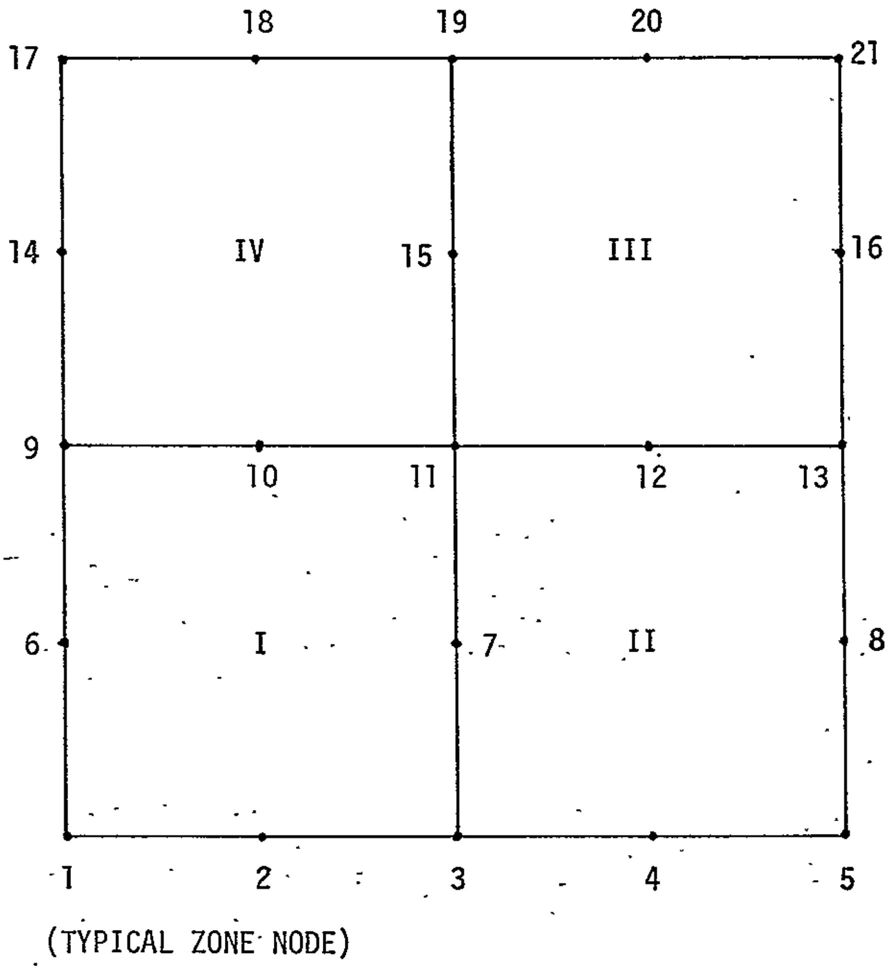


Figure A2. Simple Structure to Illustrate Zone Node Input Data.

APPENDIX B

FORTRAN LISTING OF MESH GENERATION PROGRAM, FEMESH

(LRC, CDC-6600 COMPUTER VERSION)

```
              PROGRAM FEMESH(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,PUNCH)
        C
        C
        C
        C     PROGRAM FEMESH                 CODED BY L. C. LANE          MAY 31, 1975
        C
        C
        C     PROGRAM FEMESH GENERATES FINITE ELEMENT DATA FOR TWO DIMENSIONAL
        C     PLANAR SURFACES.  STRUCTURES MAY BE SUBDIVIDED INTO AS MANY AS 15
        C     QUADRILATERAL ZONES.  THE MAXIMUM MESH DIMENSION IS 24 X 24 SUBDIVISIONS.
        C
        C
        C
        C     * NEGATIVE ZONE NODE NUMBERS FOR A ZONE IDENTIFIED BY A NUMBER **N**
        C       IDNICATES THAT THE NEGATIVE NODE IS CONNECTED TO A ZONE WHICH IS
        C       IDENTIFIED BY A NUMBER LESS THAN **N**.
        C
        C
        C     **   INODE FIRST NODE NUMBER          ** M,N  NUMBER OF SUBDIVISIONS
        C     **    ICTZ  CURRENT ZONE  NUMBER      **  NT  TOTAL NUMBER OF INPUT NODES
        C     **      IZ  TOTAL NUMBER OF ZONES     **  NI  NUMBER OF NODES TO BE READ
        C     **      IP  PUNCH WHEN IP=1           **
        C
        C
        C
000003        DIMENSION A(8),TITLE(10),XNODE(8),YNODE(8)
000003        DIMENSION X1(78),Y1(78)
000003        COMMON ICTZ,IZONE(15,10),NODE(15,25,25),TEMP(25)
000003        COMMON X2(4002),Y2(4002)
        C
        C
000003        CALL PSEUDO
000004        CALL LEROY
        C
        C     * DETERMINE INPUT - OUTPUT DEVICES
000005        IN = 5
000006        IOUT = 6
        C
        C
000007      1 FORMAT(6I5)
000007      2 FORMAT(10I5)
000007      3 FORMAT(I5,2F10.5)
```

```
000006        3 FORMAT(I5,2F10.5)
000006        4 FORMAT(1H ,12HNO. OF ZONES,I3,//)
000006        5 FORMAT(1H ,I3,1X,8I5,1X,2I4)
000006        6 FORMAT(1H ,I3,2X,F7.3,2X,F7.3)
000006        7 FORMAT(10A4)
000006        8 FORMAT(1H1,10A4///)
000006        9 FORMAT(1H ,4HZONE,15X,10HZONE NODES,18X,1HM,4X,1HN,/2X,3HNO.,4X,1H
               11,4X,1H2,4X,1H3,4X,1H4,4X,1H5,4X,1H6,4X,1H7,4X,1H8/)
000006       12 FORMAT(///,1X,4HNODE,4X,1HX,8X,1HY)
           C
           C
           C
           C       INITIALIZE X1 AND Y1 TO BE FILLED FROM DATA READ OFF CARDS
000006             DO 34 I=1,78
000010             X1(I) = 0.
000011             Y1(I) = 0.
000012       34 CONTINUE
           C
           C
           C
           C       * READ INPUT
000014             READ(IN,7)TITLE
000021             READ(IN,1)IZ,NT,NI,INODE,IELM,IP
000041             READ(IN,2)((IZONE(I,J),J=1,10),I=1,IZ)
           C
           C       * WRITE INPUT
000060             WRITE(IOUT,8)TITLE
000066             WRITE(IOUT,4)IZ
000074             WRITE(IOUT,9)
000100             WRITE(IOUT,5)(I,(IZONE(I,J),J=1,10),I=1,IZ)
000121             WRITE(IOUT,12)
           C
000125             DO 10 J=1,NI
000127             READ(IN,3) I,X1(I),Y1(I)
000140             WRITE(IOUT,6) I,X1(I),Y1(I)
000152       10 CONTINUE
           C
           C       * SET COUNTER OF NODE NUMBERS, ICTN AND ZONE,ICTZ
000155             ICTZ = 0
000156             ICTN = INODE
000157             NCOR = 0
           C
```

```
      DO 35 I=1,1661
      X2(I) = 0.
      Y2(I) = 0.
   35 CONTINUE
      DO 1010 I=1,15
      DO 1009 J=1,25
      DO 1008 K=1,25
      NODE(I,J,K) = 0
 1008 CONTINUE
 1009 CONTINUE
 1010 CONTINUE
C
C
 1000 CONTINUE
      ICTZ = ICTZ + 1
C
C
C     SET IDNICATORS TO ZERO
C
      IS1 = 0
      IS2 = 0
      IS3 = 0
      IS4 = 0
C
C     * PULL FROM THE IZONE ARRAY  THE  ZONE NUMBERS AND THE ZONE COORDINATES
C
      DO 20 I=1,8
      IC = IABS(IZONE(ICTZ,I))
      XNODE(I) = X1(IC)
      YNODE(I) = Y1(IC)
   20 CONTINUE
      M = IZONE(ICTZ,9)
      N = IZONE(ICTZ,10)
      NN = N +1
      MM = M +1
C     *
C
C
C     * TITLE THE OUTPUT FOR THIS ZONE
C
      WRITE(IOUT,11)ICTZ
   11 FORMAT(1H1,28HCALCULATIONS FOR ZONE NUMBER,I4///)
```

```
                 C
                 C        IF NO VALUE IS GIVEN FOR MIDPOINTS, ASSUME A STRAIGHT LINE AND CALCULATE
                 C        THE MIDPOINT.
                 C
000237                    DO 30 I=2,8,2
000241                    IF(XNODE(I))30,25,30
000242                 25 IF(YNODE(I))30,26,30
000244                 26 K = 8-I
000246                    IF(K)30,27,28
000247                 28 XNODE(I) = (XNODE(I+1)+XNODE(I-1))/2.
000253                    YNODE(I) = (YNODE(I+1)+YNODE(I-1))/2.
000256                    GO TO 30
000256                 27 XNODE(I) = (XNODE(1)+XNODE(7))/2.
000262                    YNODE(I) = (YNODE(1)+YNODE(7))/2.
000264                 30 CONTINUE
000266                    WRITE(IOUT,31)(I,XNODE(I),YNODE(I),I=1,8)
000303                 31 FORMAT(1H ,10HZONE NODE ,I4,F15.2,F25.2)
000303                    WRITE(IOUT,32)
000307                 32 FORMAT(//,1X,8HNODE NO.,5X,1HX,8X,1HY/)
                 C
                 C
                 C
                 C
                 C        * IF ZONE NUMBER IS ONE,FILL NODE ARRAY AND SKIP TO X, Y COORDINATE
                 C             CALCULATIONS.
                 C
000307                    IF(ICTZ-1)199,190,201
000312                190 DO 192 J=1,NN
000314                    DO 191 I=1,MM
000315                    NODE(1,I,J) = ICTN
000322                    ICTN = ICTN + 1
000323                191 CONTINUE
000325                192 CONTINUE
000330                    GO TO 3000
000330                199 WRITE(IOUT,200)ICTZ
000336                200 FORMAT(1H1,28HERROR ... ZONE NUMBER ICTZ =.I4)
000336                201 CONTINUE


                          DETERMINE WHICH SIDES ARE CONNECTED
                          FILL THE NODE ARRAY
```

```
        C
        C     SIDE ONE
        C
000336        IF(IZONE(ICTZ,2))212,220,220
000340    212 CALL FIND(1,3,1)
000343        DO 213 I=1,MM
000345        NODE(ICTZ,I,1) = TEMP(I)
000353    213 CONTINUE
000355        IS1 = 1
        C
        C
        C     SIDE 2
        C
000356    220 CONTINUE
000356        IF(IZONE(ICTZ,4))222,230,230
000360    222 CALL FIND(3,5,2)
000363        DO 223 I=1,NN
000365        NODE(ICTZ,MM,I) = TEMP(I)
000375    223 CONTINUE
000377        IS2 = 1
        C
        C
        C     SIDE 3
        C
000400    230 CONTINUE
000400        IF(IZONE(ICTZ,6))232,240,240
000402    232 CALL FIND(5,7,3)
000405        DO 233 I=1,MM
000407        NODE(ICTZ,I,NN) = TEMP(I)
000417    233 CONTINUE
000421        IS3 = 1
        C
        C
        C     SIDE 4
        C
000422    240 CONTINUE
000422        IF(IZONE(ICTZ,8))242,250,250
000424    242 CALL FIND(7,1,4)
000427        DO 243 I=1,NN
000431        NODE(ICTZ,1,I) = TEMP(I)
000437    243 CONTINUE
000441        IS4 = 1
000442    250 CONTINUE
```

```
           C
           C
           C
           C      FILL NODE ARRAY - JUMP THOSE POSITIONS ALL READY FILLED
           C
           C
000442            DO 320 J=1,NN
000444            DO 310 I=1,MM
000445            IF(NODE(ICTZ,I,J))315,300,310
000453        300 CONTINUE
000453            NODE(ICTZ,I,J) = ICTN
000461            ICTN = ICTN + 1
000462            GO TO 310
           C      ERROR
000463        315 CONTINUE
000463            WRITE(IOUT,316)
000467        316 FORMAT(48H NODE NO FOUND IN ST. N/. 300-320 LESS THAN ZERO)
000467        310 CONTINUE
000472        320 CONTINUE
000474       3000 CONTINUE
           C
           C
           C      * COMPUTE THE X-Y COORDINATES, OMITT PREVIOUSLY COMPUTED SITES.
           C
           C      * DC, AND DN ARE THE INCREMENTAL VALUES IN THE M AND N
           C          DIRECTIONS RESPECTIVELY.
000474            RM = M
000476            RN = N
000477            DC = 2./RM
000501            DN = 2./RN
           C      WRITE(IOUT,33)DC,DN
           C   33 FORMAT(1H ,4HDC= ,F5.2,4X,4HDN= ,F5.2)
           C
000503            CCC = -1.
000504            CCN = -1.
           C
000505            DO 810 J=1,NN
000506            IF(J-1)731,730,731
000510        730 IF(IS1-1)733,51,733
000513        731 IF(J-NN)733,732,733
000515        732 IF(IS3-1)733,51,733
000517        733 CONTINUE
           C
```

```
000517          CO 800 I=1,MM
        C
000521          IF(I-MM)741,734,741
000523      734 IF(IS2-1)739,50,739
        C
000526      741 IF(I-1)739,742,739
000530      742 IF(IS4-1)739,50,739
000532      739 CONTINUE
000532          S1 = 1.-CCC
000534          S2 = 1.-CCN
000536          S3 = CCC+CCN-1.
000540          S4 = 1.+CCC
000541          S5 = 1.+CCN
000543          A(1) =  1./4.*S1*S2*(-CCC-CCN-1.)
000552          A(2) = 1./2.*S1*S2*S4
000554          A(3) =  1./4.*S2*S4*(CCC-CCN-1.)
000563          A(4) = 1./2.*S4*S2*S5
000566          A(5) =  1./4.*S3*S4*S5
000571          A(6) =  1./2.*S1*S4*S5
000575          A(7) =  1./4.*S1*S5*(-CCC+CCN-1.)
000604          A(8) = 1./2.*S1*S2*S5
000610          NCOR = NCOR + 1
000611          CO 45 K=1,8
000613          X2(NCOR) = A(K) * XNODE(K) + X2(NCOR)
000617          Y2(NCOR) = A(K) * YNODE(K) + Y2(NCOR)
000622       45 CONTINUE
000624          WRITE(IOUT,2000) NCOR,X2(NCOR),Y2(NCOR)
000635     2000 FORMAT(I5,2X,F10.5,2X,F10.5)
000635       50 CONTINUE
000635          CCC = CCC+DC
000637      800 CONTINUE
000642       51 CONTINUE
000642          CCN = CCN + DN
000644          CCC = -1.
000646      810 CONTINUE
000650          WRITE(IOUT,54)
000654       54 FORMAT(///,1H ,15HNODE NO. MATRIX/)
000654          CO 53 J=1,NN
000656          WRITE(IOUT,52)(NODE(ICTZ,I,J),I=1,MM)
000673       52 FORMAT(1X,26I5)
000673       53 CONTINUE
000676          IF(ICTZ-IZ)1000,40000,40000
000700    40000 CONTINUE
```

```
            C
            C      * LIST THE ELEMENT NUMBERS AND DEFINING NODE NUMBERS.
            C
            C
000700             WRITE(IOUT,900)
000704         900 FORMAT(1H1,7HELEMENT,8X,1HI,10X,1HJ,10X,1HK,10X,1HL/)
000704             DO 930 ICTZ=1,IZ
000706             M = IZONE(ICTZ,9)
000710             N = IZONE(ICTZ,10)
000711             DO 920 J=1,N
000713             DO 910 I=1,M
000714             II = I + 1
000716             JJ = J + 1
000717             INO = NODE(ICTZ,I,J)
000724             JNO = NODE(ICTZ,II,J)
000731             KNO = NODE(ICTZ,II,JJ)
000735             LNO = NODE(ICTZ,I,JJ)
000742             WRITE(IOUT,901)IELM,INO,JNO,KNO,LNO
            C
            C      PUNCHED OUTPUT IN FORMAT FOR USE IN PROGRAM SAP
000757             IF(IP .EQ. 1) PUNCH 902, IELM,INO,JNO,KNO,LNO
            C
000777         901 FORMAT(I6,4(6X,I5))
000777         902 FORMAT(5I5)
000777             IELM = IELM + 1
001001         910 CONTINUE
001003         920 CONTINUE
001006         930 CONTINUE
            C
001010             CALL FEMPLT(IZ,INODE,NCOR)
            C
            C      * WRITE THE X AND Y COORDINATES OF THE NODE NUMBERS.
            C
001013             WRITE(IOUT,940)
001017         940 FORMAT(1H1,4HNODE,11X,1HX,14X,1HY/)
001017             DO 950 I=1,NCOR
001021             WRITE(IOUT,941) INODE,X2(I),Y2(I)
001032         941 FORMAT(1H ,I5,2(5X,F10.4))
001032             INODE = INODE + 1
001034         950 CONTINUE
            C      *
001036             STOP
001040             END
```

```
                  SUBROUTINE FIND(LP1,LP2,ISD)
            C
            C     SEARCHES PREVIOUSLY CALCULATED DATA TO FIND NODE NUMBERS ASSIGNED TO A
            C     ZONE BOUNDARY.
            C
000006            COMMON ICTZ,IZONE(15,10),NODE(15,25,25),TEMP(25)
000006            COMMON X2(4002),Y2(4002)
            C
            C     LP      LOCATION OF PT..ON ZONE ICTZ
            C
000006            IA = 0
000007            IT = 0
            C
000010            DO 5 I=1,25
000011            TEMP(I) = 9999
000013          5 CONTINUE
            C
            C     DEFINE CORNOR NODES ON ZONE ICTZ
            C
000015            J1 = IABS(IZONE(ICTZ,LP1))
000021            J2 = IABS(IZONE(ICTZ,LP2))
            C
000024            IIZ = ICTZ
000025         10 IIZ = IIZ - 1
000027            IF(IIZ)200,200,110
000030        110 CONTINUE
            C
            C     SEARCH DATA OF ZONE IIZ
            C
000030            DO 40 I=1,7,2
000032            II = I + 2
000034            IF(I-7)16,15,16
000036         15 II = 1
000037         16 CONTINUE
000037            K1 = IABS(IZONE(IIZ,I))
000043            K2 = IABS(IZONE(IIZ,II))
            C
            C     COMPARE ICTZ TO IIZ
            C
000047            IF(J1-K1)30,20,30
000051         20 IF(J2-K2)40,21,40
000054         30 IF(J1-K2)40,31,40
```

```
000056        31 IF(J2-K1)40,21,40
000060        21 CONTINUE
000060           IA = (I+1)/2
000063           GO TO 41
000063        40 CONTINUE
           C
000065        41 CONTINUE
000065           IF(IA)45,10,45
           C
           C
           C     PUT DESIRED CONTENTS OF IIZ IN TEMPERORY ARRAY
           C
           C
           C     TEMP ARRAY MUST HAVE REVERSE ORDER IF ...
           C              ISD = 1 OR 2    AND    IA = 1 OR 2
           C                         - OR -
           C              ISD = 3 OR 4    AND    IA = 3 OR 4
           C
           C
000066        45 CONTINUE
000066           MMT = IZONE(IIZ,9) + 1
000071           NNT = IZONE(IIZ,10) + 1
000072           MK = MMT
000074           NK = NNT
000075           II = 0
           C
000076           DO 100 I=1,25
           C
000077           IF(IA-1)46,50,46
000101        46 IF(IA-2)47,60,47
000103        47 IF(IA-3)48,70,48
000105        48 IF(IA-4)100,80,100
           C
000110        50 CONTINUE
000110           MK = I
000112           NK = 1
000113           IF(ISD-2)90,90,92
           C
000116        60 CONTINUE
000116           NK = I
000120           IF(ISD-2)91,91,92
           C
000123        70 CONTINUE
```

```
000123          MK = I
000125          IF(ISD-2)92,92,9C
       C
0C0130       80 CONTINUE
000130          MK = 1
0C0131          NK = I
000133          IF(ISC-2)92,92,91
       C
       C
000136       90 II = MMT + 1 - I
0C0141          GO TO 93
000141       91 II = NNT + 1 - I
000144          GO TO 93
0C0144       92 II = I
000146       93 CONTINUE
000146          IF(II)200,200,94
000150       94 CONTINUE
       C
000150          TEMP (II) = NODE(IIZ,MK,NK)
000157      100 CONTINUE
0C0161      200 CONTINUE
000161          RETURN
000162          END
```

```
              SUBROUTINE FEMPLT(IZ,INODE,NCOR)
        C
        C     PLOTS THE FINITE ELEMENT MESH
        C
        C
000006        COMMON ICTZ,IZONE(15,10),NODE(15,25,25),TEMP(25)
000006        COMMON X2(4002),Y2(4002)
        C
        C
        C     SCALE DATA
000006        CALL ASCALE(X2,25.,NCOR,1,20.)
000011        CALL ASCALE(Y2,13.,NCOR,1,20.)
        C
000016        XSCALE = X2(NCOR + 2)
000022        YSCALE = Y2(NCOR + 2)
        C
000023        IF(XSCALE .GE. YSCALE) SF=XSCALE
000027        IF(YSCALE .GE. XSCALE) SF=YSCALE
        C
        C
000032        DO 100 ICTZ=1,IZ
000034        M = IZONE(ICTZ,9)
000036        N = IZONE(ICTZ,10)
        C
000037        DO 90 J=1,N
000041        DO 80 I=1,M
        C
000042        II = I + 1
000044        JJ = J + 1
        C
        C     DEFINE THE 4 NODES OF AN ELEMENT
        C
000045        INO = NODE(ICTZ,I,J) - INODE + 1
000053        JNO = NODE(ICTZ,II,J) - INODE + 1
000061        KNO = NODE(ICTZ,II,JJ) - INODE + 1
000067        LNO = NODE(ICTZ,I,JJ) - INODE + 1
        C
        C     DEFINE THE X AND Y COORDINATES OF THE 4 NODES
        C
000075        XI = X2(INO)/SF
000100        XJ = X2(JNO)/SF
000102        XK = X2(KNO)/SF
```

```
000104            XL = X2(LNO)/SF
            C
000106            YI = Y2(INO)/SF
000110            YJ = Y2(JNO)/SF
000112            YK = Y2(KNO)/SF
000115            YL = Y2(LNO)/SF
            C
            C     PLOT THE 4 NODES.
            C
000117            CALL CALPLT(XI,YI,3)
000121            CALL CALPLT(XJ,YJ,2)
000124            CALL CALPLT(XK,YK,2)
000127            CALL CALPLT(XL,YL,2)
000132            CALL CALPLT(XI,YI,2)
            C
000135         80 CONTINUE
000142         90 CONTINUE
000144        100 CONTINUE
000147            RETURN
000150            END
```

REFERENCES

1. W.R. Buell and B.A. Bush, "Mesh Generation - A Survey" an ASME publication, Paper No. 73-WA/DEZ.

2. O.C. Zienkiewicz and D.R. Phillips, "An Automatic Mesh Generation Scheme for Plane and Curved Surfaces by 'Isoparametric' Coordinates", International Journal for Numerical Methods in Engineering, Vol. 3, 519-528, 1974.

3. S.J. Womack, "Shape Function Techniques for Generation of Finite Element Grids", TICOM Report 73-3, August 1973.

4. R.S. Dunham and E.B. Becker, "TEXGAP - The Texas Grain Analysis Program", TICOM Report 73-1, August 1973.

5. O.C. Zienkiewicz, The Finite Element Method in Engineering Science, McGraw-Hill, 1971.

Table 1. The shape functions.

$$x = \sum_{i=1}^{8} N_i \, x_i$$

$$y = \sum_{i=1}^{8} N_i \, y_i$$

$$N_1 = -\frac{1}{4} (1 - \xi) (1 - \eta) (\xi + \eta + 1)$$

$$N_2 = \frac{1}{2} (1 - \xi^2) (1 - \eta)$$

$$N_3 = \frac{1}{4} (1 + \xi) (1 - \eta) (\xi - \eta - 1)$$

$$N_4 = \frac{1}{2} (1 + \xi) (1 - \eta^2)$$

$$N_5 = \frac{1}{4} (1 + \xi) (1 + \eta) (\xi + \eta - 1)$$

$$N_6 = \frac{1}{2} (1 - \xi^2) (1 + \eta)$$

$$N_7 = \frac{1}{4} (1 - \xi) (1 + \eta) (-\xi + \eta - 1)$$

$$N_8 = \frac{1}{2} (1 - \xi) (1 - \eta^2)$$

Table 2. Input Data for Example Problem Shown in Figure 5.

```
EXAMPLE PROBLEM
  3    18     8   100 1000
  1     2     3     7   11   10    9    6    3    3
 -3     4     5     8   13   12  -11   -7    2    3
-11   -12   -13    15   18   17   16   14    2    4
  1          0.          0.
  3          3.          0.
  5          5.          0.
  9          0.          3.
 11          3.          3.
 13          5.          3.
 16          3.          7.
 18          5.          7.
```

C-8

Table 3. Output for Example Problem Shown in Figure 5.

EXAMPLE PROBLEM

NO. OF ZONES 3

ZONE NO.	ZONE NODES 1	2	3	4	5	6	7	8	M	N
1	1	2	3	7	11	10	9	6	3	3
2	-3	4	5	8	13	12	-11	-7	2	3
3	-11	-12	-13	15	18	17	16	14	2	4

NODE	X	Y
1	0.000	0.000
3	3.000	0.000
5	5.000	0.000
9	0.000	3.000
11	3.000	3.000
13	5.000	3.000
16	3.000	7.000
18	5.000	7.000

(cont'd.)

Table 3. Output for Example Problem Shown in Figure 5 (continued).

ELEMENT	I	J	K	L
1000	100	101	105	104
1001	101	102	106	105
1002	102	103	107	106
1003	104	105	109	108
1004	105	106	110	109
1005	106	107	111	110
1006	108	109	113	112
1007	109	110	114	113
1008	110	111	115	114
1009	103	116	118	107
1010	116	117	119	118
1011	107	118	120	111
1012	118	119	121	120
1013	111	120	122	115
1014	120	121	123	122
1015	115	122	125	124
1016	122	123	126	125
1017	124	125	128	127
1018	125	126	129	128
1019	127	128	131	130
1020	128	129	132	131
1021	130	131	134	133
1022	131	132	135	134

(cont'd.)

Table 3. Output for Example Shown in Figure 5 (concluded).

NODE	X	Y
100	0.0000	0.0000
101	1.0000	0.0000
102	2.0000	0.0000
103	3.0000	0.0000
104	0.0000	1.0000
105	1.0000	1.0000
106	2.0000	1.0000
107	3.0000	1.0000
108	0.0000	2.0000
109	1.0000	2.0000
110	2.0000	2.0000
111	3.0000	2.0000
112	0.0000	3.0000
113	1.0000	3.0000
114	2.0000	3.0000
115	3.0000	3.0000
116	4.0000	0.0000
117	5.0000	0.0000
118	4.0000	1.0000
119	5.0000	1.0000
120	4.0000	2.0000
121	5.0000	2.0000
122	4.0000	3.0000
123	5.0000	3.0000
124	3.0000	4.0000
125	4.0000	4.0000
126	5.0000	4.0000
127	3.0000	5.0000
128	4.0000	5.0000
129	5.0000	5.0000
130	3.0000	6.0000
131	4.0000	6.0000
132	5.0000	6.0000
133	3.0000	7.0000
134	4.0000	7.0000
135	5.0000	7.0000

Table 4. Input for Shear Panel.

```
 SHEAR PANEL
  4   21   17    1 1000
  1    2    3    4    5    8   11   10   20   10
-11   -8   -5    6    7    9   13   12   20    3
 -1  -10  -11   15   19   18   17   14   10   20
-11  -12  -13   16   21   20  -19  -15    3   20
  1  0.        0.
  2 3.0        0.
  3 4.35       0.
  4 4.3727     .1040
  5 4.435      .185
  7 4.6        .35
  8 3.825      .825
  9 3.975      .975
 11 2.31       2.31
 13 2.475      2.475
 14 0.         3.
 15 .825       3.825
 16 .975       3.975
 17 0.         4.35
 18 .1040      4.3727
 19 .185       4.435
 21 .350       4.6
```

Table 5. Input for Bolted Joint Specimen.

```
BOLTED JOINT COMPOSITE STRUCTURE
 15    66    41   100 1000
 28    29    30    21     3     2     1    20     2     3
-30    22     7     6     5     4    -3   -21     5     3
 -7   -22   -30    31    32    23     9     8     5     6
 -9   -23   -32    24    13    12    11    10     5     3
-30    41    57    56    55    42   -32   -31     5     6
 49    50    51    58   -57   -41   -30    40     5     6
 47    48   -49   -40   -30   -29   -28    39     2     6
-51   -50   -49    60    61    59    53    52     5     6
-53   -59   -61    43   -32   -42   -55    54     5     6
-61    62    63    44    34    33   -32   -43     3     6
-32   -33   -34    25    15    14   -13   -24     3     3
-34    35    36    26    17    16   -15   -25    11     3
-36    37    38    27    19    18   -17   -26     6     3
-63    64    65    45   -36   -35   -34   -44    11     6
-65    66    67    46   -38   -37   -36   -45     6     6
  1  0.        1.5
  3  0.13      1.5
  4 0.4290     1.5
  5  0.5345    1.5
  6  0.5418    1.4635
  7  0.5625    1.4325
  8  0.63      1.4045
  9  0.6975    1.4325
 10  0.7182    1.4635
 11 0.7255     1.5
 12 0.8308     1.5
 13 1.13       1.5
 15 1.5        1.5
 17 3.3        1.5
 19 4.5        1.5
 22 0.4292     1.284
 23 0.8308     1.284
 28 0.         1.0
 30 0.13       1.0
 32 1.13       1.0
 34 1.5        1.0
 36 3.3        1.0
 38 4.5        1.0
 41 0.4292     0.7008
 42 0.8308     0.7008
 47 0.         0.
 49 0.13       0.
 50 0.4292     0.2992
 51 0.5625     0.4325
 52 0.63       0.4045
 53 0.6975     0.4325
 54 0.7255     0.5
 55 0.6975     0.5675
 56 0.63       0.5955
 57 0.5625     0.5675
 58 0.5345     0.5
 59 0.8308     0.2992
 61 1.13       0.
 63 1.5        0.
 65 3.3        0.
 67 4.5        0.
```

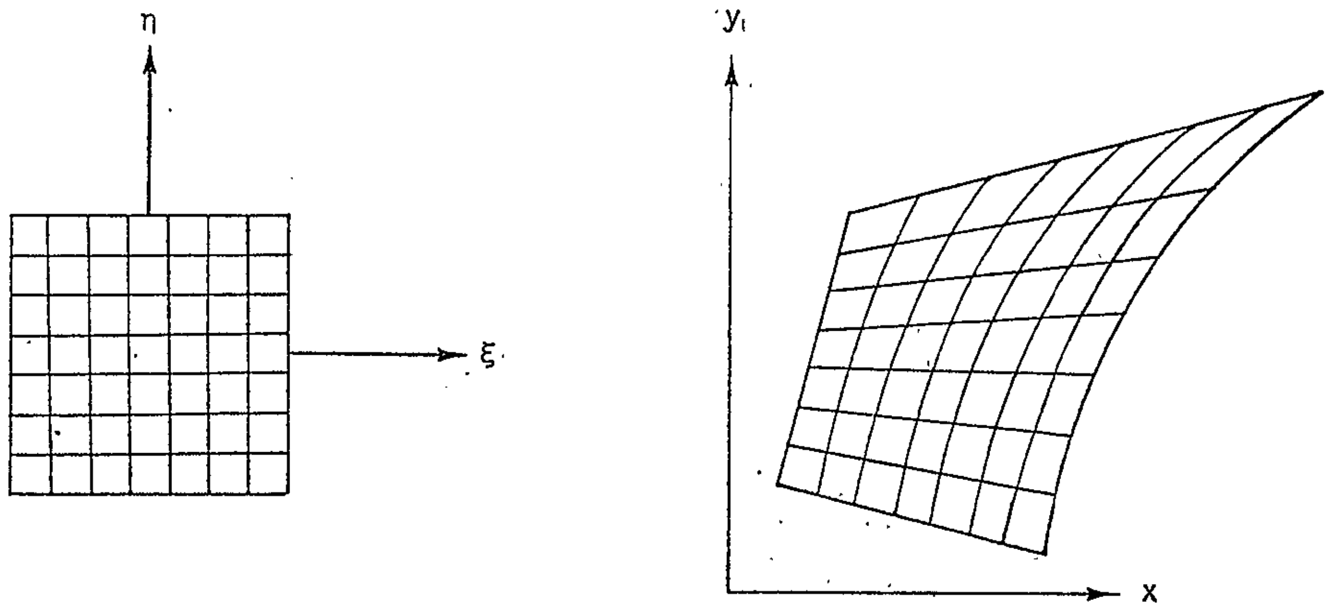
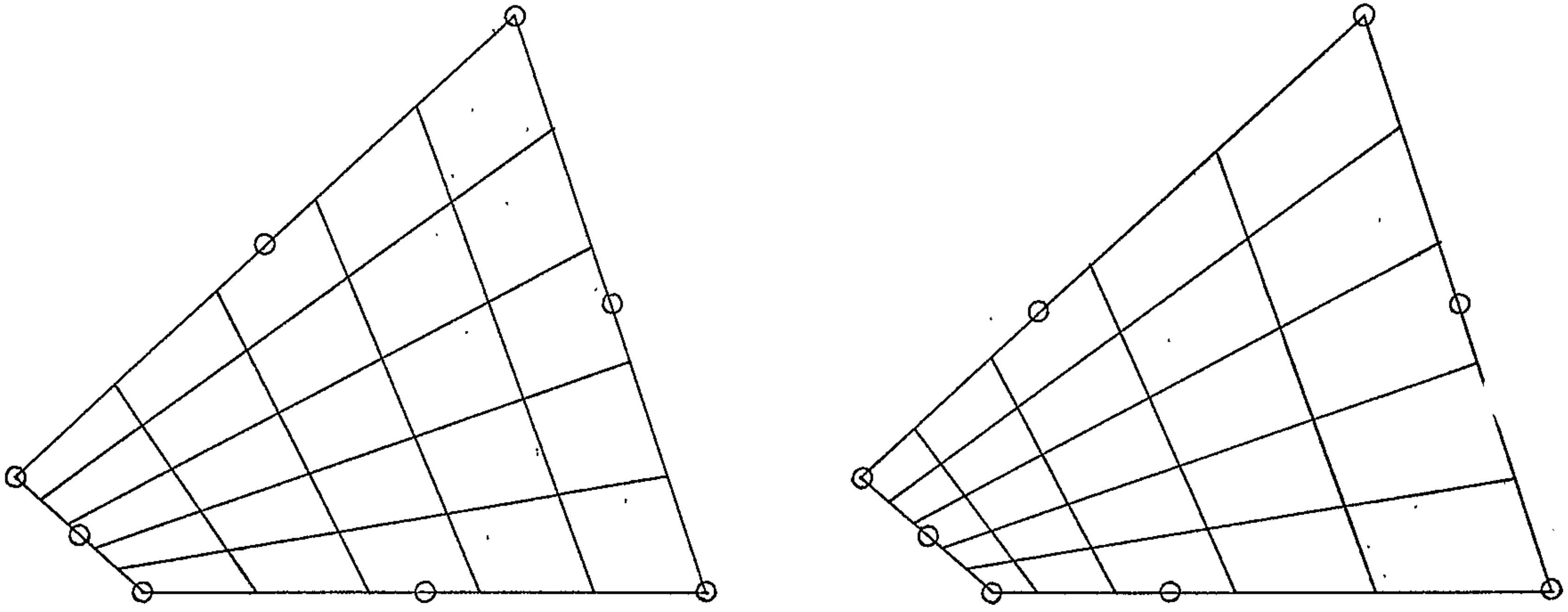


Figure 1. Mapping of a Quadrilateral from the Natural to the Cartesian Coordinate System.



(a) UNGRADED MESH

(b) GRADED MESH

Figure 2. Ungraded and Graded Mesh Generated in the Cartesian System.

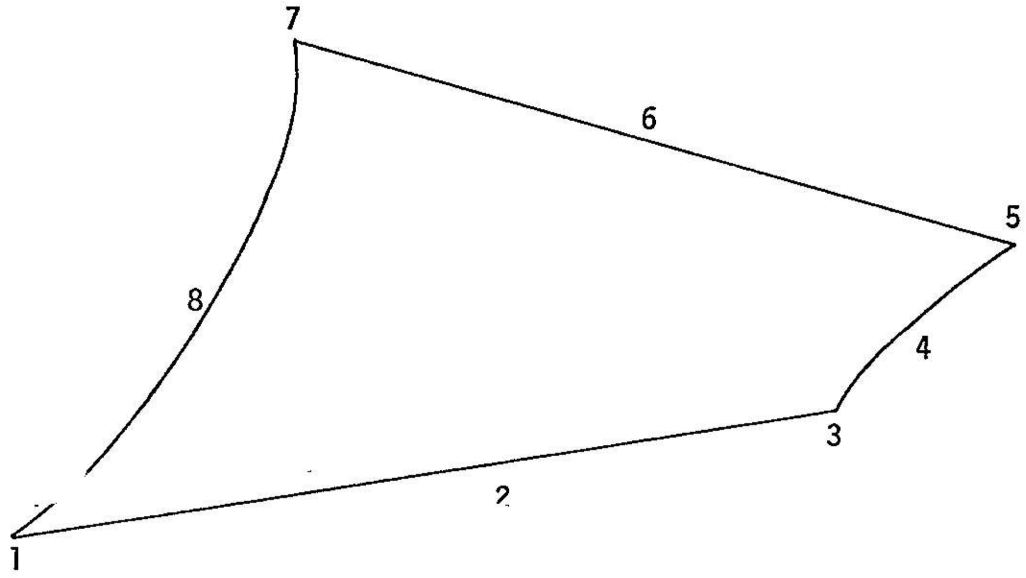


Figure 3. Numbering of Zone Nodes.

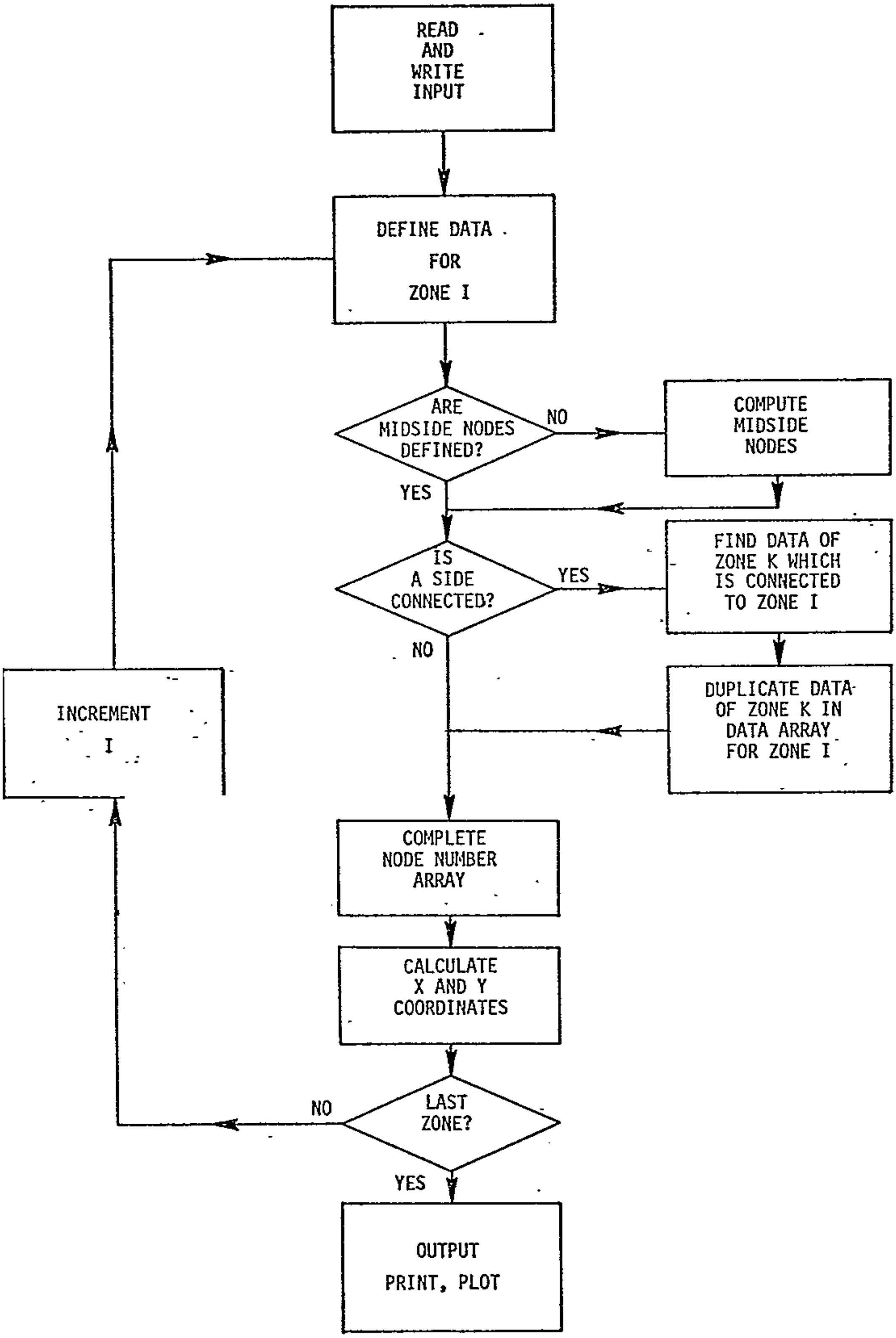


Figure 4. Flowchart for Mesh Generation Program, FEMESH.

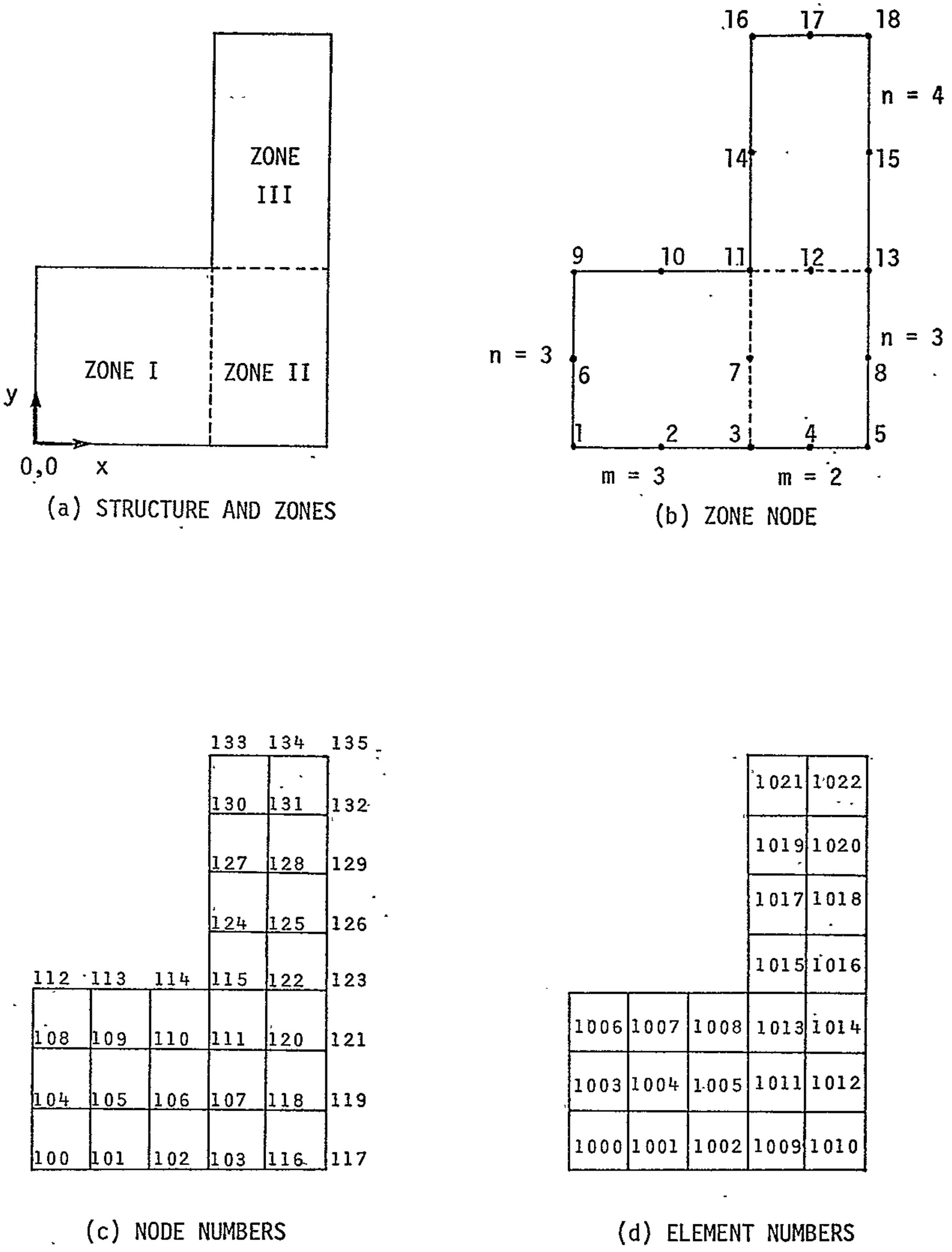


Figure 5. Example of Mesh Generation for a Simple Structure.

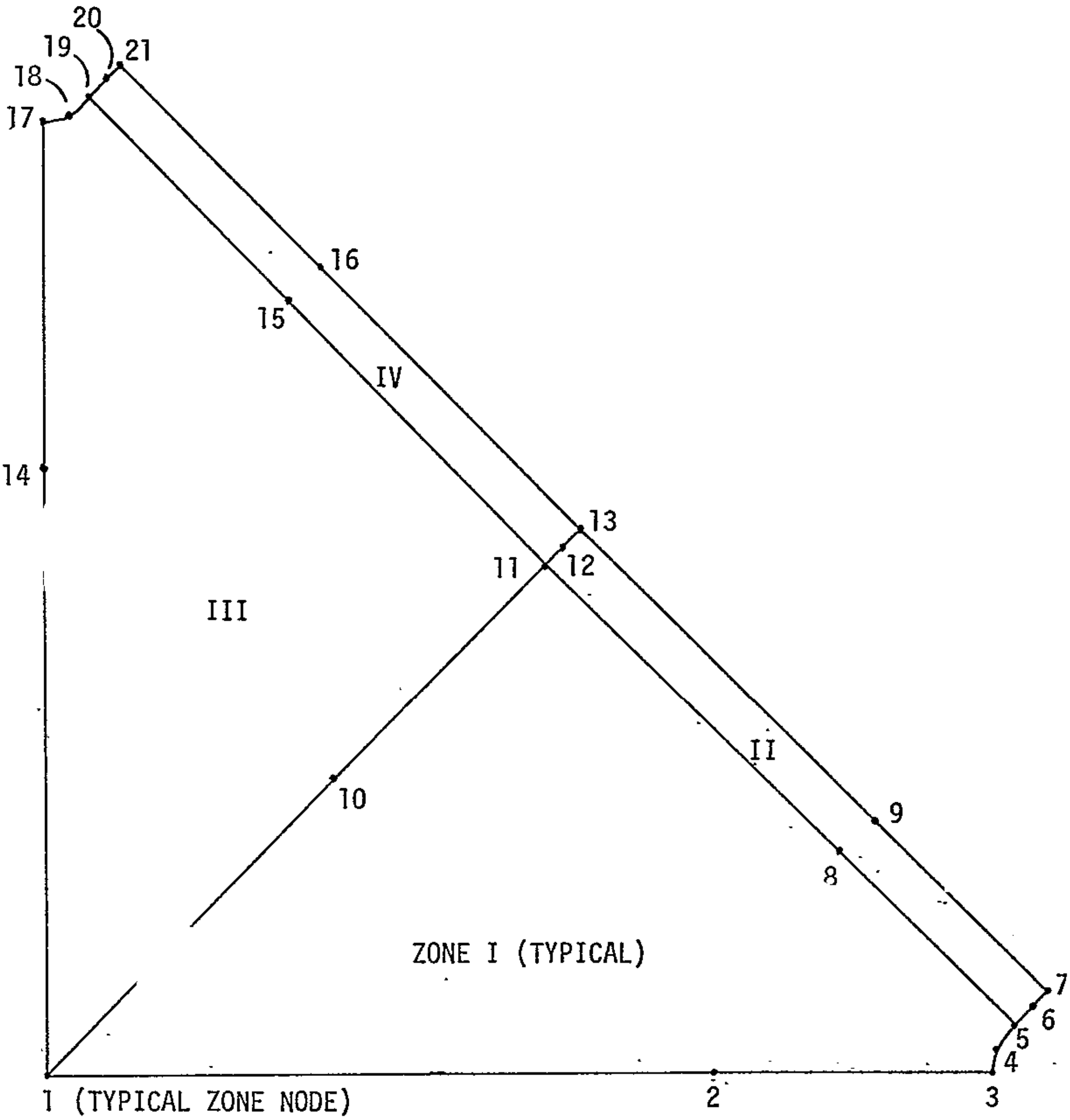


Figure 6. Zones of Shear Panel.

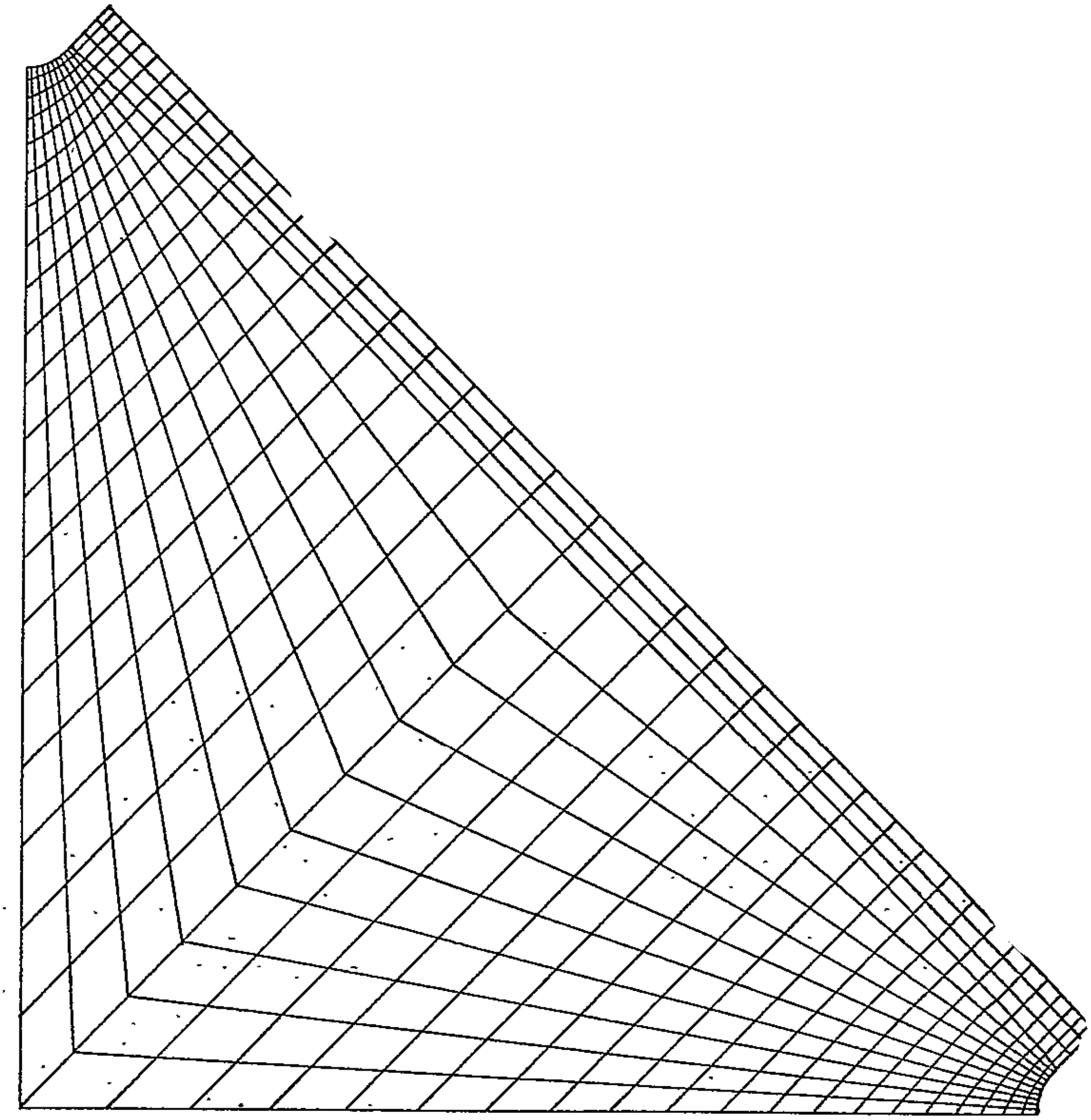


Figure 7. Mesh Generated for Quartersection of Shear Panel.

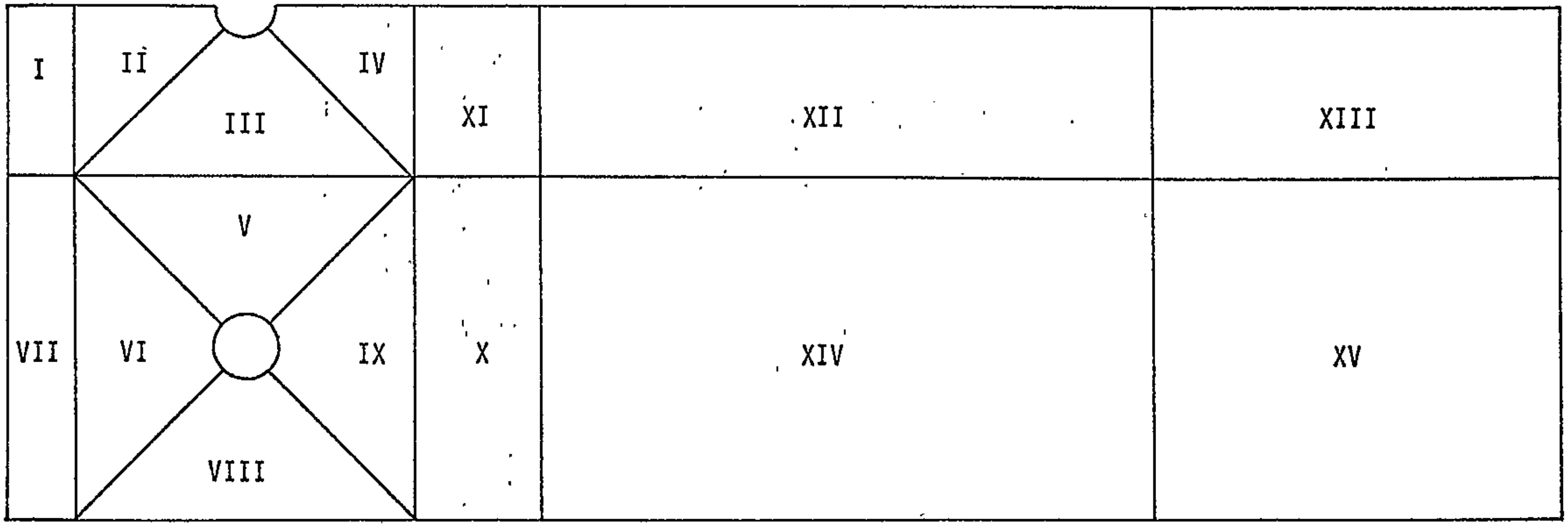


Figure 8. Zones of Bolted Joint Specimen.

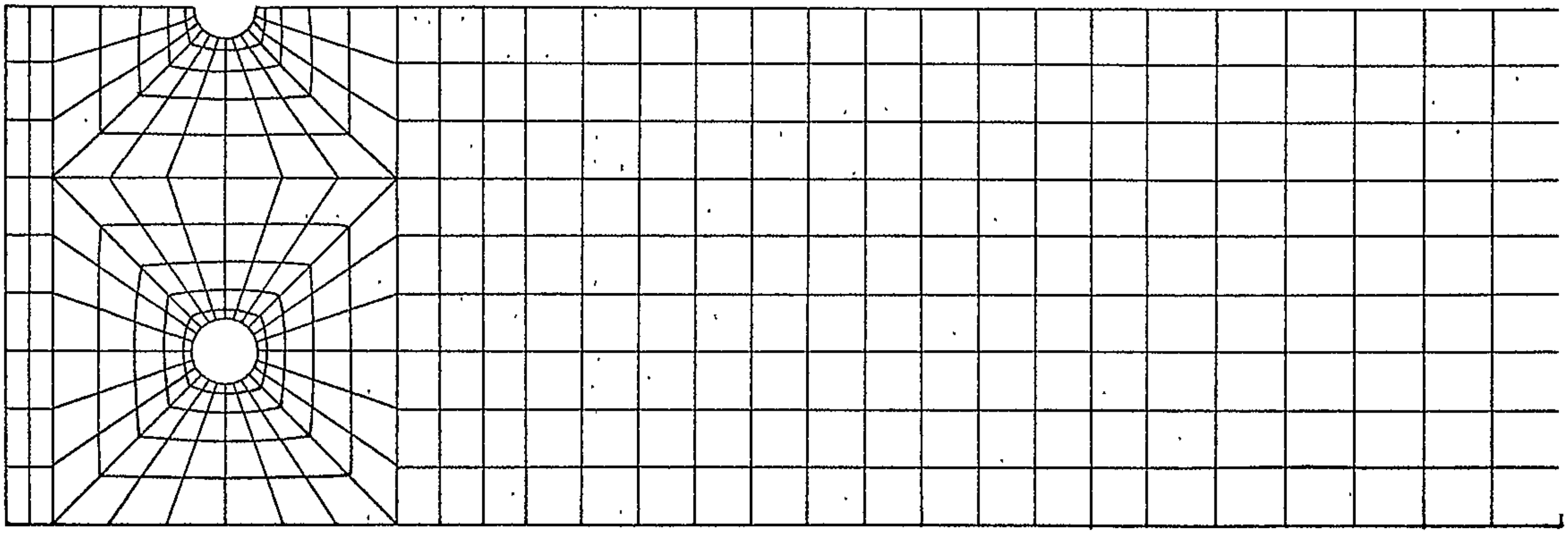


Figure 9. Mesh Generated for Half Section of Bolted Joint Specimen.

FINITE ELEMENT ANALYSIS OF A COMPOSITE BOLTED JOINT SPECIMEN

By

Earl A. Thornton

FINITE ELEMENT ANALYSIS OF A COMPOSITE BOLTED JOINT SPECIMEN

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INTRODUCTION

With high strength and weight savings, advanced composite materials have become increasingly important in aircraft structural design. The full potential for the increase of structural efficiency through the use of advanced composites has not yet been fully realized because of low efficiencies in mechanical joints. The Advanced Composites Design Guide (ref. 1) states that weight savings may be reduced by as much as 40 percent due to such practical constraints.

In the use of conventional materials, design methods for joints have evolved over a period of time from data gathered from experimental and analytical solutions and, in addition, are often based upon rules-of-thumb derived from experience. For advanced composites, such data and experience are relatively limited. To partially fill this need, test programs are underway at Langley Research Center (LRC) to establish data on a number of mechanical joint designs (ref. 2)

The purpose of the present study was to provide analytical support for the LRC bolted joint test program. Specific objectives of the study were to: (1) determine the laminate stress distribution in an extra graphite reinforced bolted joint specimen, and (2) compare two methods of modeling bolt transfer loads for determination of stress distributions in bolted joints.

This paper will describe the finite element model used to represent the bolted joint specimen. The two methods used to represent bolt transfer loads will be discussed. Laminate membrane force distributions predicted by the finite element

analysis will be presented, and force gradients at the bolt holes will be discussed. Differences in the results due to the methods of representing the bolt loads will also be discussed.

BOLTED JOINT SPECIMEN

The specimen analyzed in this study is the specimen denoted as extra graphite reinforced joint specimen number one, reference 2. The specimen is shown schematically in figure 1 with the dimensions used in the analysis. The specimen was fabricated from a basic layup of 15 plies reinforced by additional plies so that in the thick section where the bolt holes are located there are 49 plies. The ply stacking sequences are shown in figure 2 with cross-sectional details of the layup. Reinforcing plies increase by 0.1 in. in length per ply over the transition section from 49 plies to 15 plies.

ANALYTICAL PROCEDURES

Finite Element Model

The NASA Structural Analysis (NASTRAN) computer program (level 15.5) was used to compute the laminate stress distributions in the specimen. The specimen was assumed to be in-plane stress and due to symmetry only one-half of the specimen was represented with finite elements. The finite element representation is shown in figure 3. The specimen was represented by an assemblage of 349 quadrilateral and triangular membrane elements. The NASTRAN finite elements used have constant stress throughout each element. The mathematical model has 307 grid points and 573 degrees of freedom. Vertical displacements were set to zero on the top boundary of the finite element model to represent symmetry, and horizontal displacements at the right edge of the finite element model were set to zero to represent clamping in the test fixture.

In the analytical formulation underlying the present NASTRAN elements the element material is assumed homogeneous through the

thickness. The element extensional stiffnesses are obtained internally in NASTRAN by multiplying the material elasticity matrix by the thickness of the element, reference 3. However, the specimen in the present study is characterized by several layers of material which are assumed homogeneous within the individual layers only. Thus for the composite laminate the extensional stiffnesses, A_{ij}, were computed externally using laminated plate theory. The stiffnesses were then input to NASTRAN in place of the material elasticity matrix, and the thickness of the specimen was everywhere taken as unity.

The extensional stiffnesses A_{ij}, a 3 x 3 symmetric matrix, were computed from reference 4:

$$A_{ij} = \sum_{k=1}^{N} (Q_{ij})_k \ (Z_k - Z_{k-1}) \tag{1}$$

where $(Q_{ij})_k$ denotes the material elasticity matrix for a single layer and $(Z_k - Z_{k-1})$ denotes the thickness of the kth layer. The extensional stiffnesses relate the in-plane membrane forces (N_x, N_y, N_{xy}) to the midplane extensional strains $(\varepsilon_x, \varepsilon_y, \gamma_{xy})$ of the laminate. Since the extensional stiffnesses were input to NASTRAN in place of the NASTRAN material elasticity matrix, the NASTRAN membrane element stresses $(\sigma_x, \sigma_y, \gamma_{xy})$ were the laminate stress resultants (N_x, N_y, N_{xy}).

In the present analysis the lamina elastic constants were taken as $E_{11} = 20 \times 10^6$ psi, $E_{22} = 2 \times 10^6$ psi, $G = 0.8 \times 10^6$ psi and $\nu_{12} = 0.3$. Each lamina had a thickness of 0.00542 in. To represent the tapered character of the specimen, extensional stiffnesses were computed for the 19 different cross-sectional layups. The values of the extensional stiffnesses for the specimen are given in table 1.

Bolt Loads

The specimen was analyzed for loading corresponding to the design failure load. This loading, estimated at 21 813 lb was assumed to be equally distributed to the three bolts such that the total load transmitted to the specimen per bolt was 7271 lb. In the finite element model, one-half of this load was applied to the center bolt hole and the full value was applied to the lower bolt hole.

Two methods were used to represent the transfer of the bolt forces to the finite element model. In the first approach the bolt was assumed to have a perfect fit, and the load transfer was assumed to take place over 180° of the bolt hole. The contact force was assumed to vary sinusoidally over this area of contact. Equilibrium of the bolt was then used to obtain the relation:

$$N = \frac{2}{\pi} \frac{2Q}{R} \cos \theta \tag{2}$$

where N denotes the contact force per unit arc length, $2Q$ is the total bolt load, and R is the radius of the bolt hole. The angle θ is measured from a horizontal axis through the hole Equation (2) was used to compute equivalent grid point forces for each grid point in the contact region ($-90° < \theta < 90°$). The equivalent grid point forces were computed by integrating Equation (2) through an angle of -6° to +6° at each grid point The equivalent grid point loads are shown in figure 4.

In the second approach an imperfect fit was assumed and a nonlinear analysis of the bolt transfer loads was made. This analysis, made using the computer program CONTACT developed in reference 5, consists of increasing the bolt load in increments and determining the number of grid points in contact and their loads at each load increment. The analysis requires as part of its input the flexibility matrix for the bolt hole. This flexibility matrix was obtained from the finite element model by applying unit loads at each node of the center bolt hole. The

16 x 16 flexibility matrix was computed one column at a time for 16 unit load subcases. This matrix was then input to the CONTACT program and the bolt transfer forces were computed for several load increments. The bolt transfer forces and the region of contact for four load increments including the maximum load are shown in figure 5. These forces were computed using an initial lack of fit of -0.00287 in. This value, as defined in the program, denotes a clearance based upon the radius of the hole.

RESULTS AND DISCUSSION

The membrane force distributions at the center and outside bolt holes as predicted by the finite element analysis are shown in figures 6 through 8. Shown are plots of the radial force N_r, the circumferential force N_θ, and the in-plane shearing force $N_{r\theta}$ versus the angle θ from the centerline. Predictions based upon the two methods of representing the bolt transfer loads are compared.

There is very little, if any, difference in the membrane forces between the center bolt hole and the outside bolt holes. Each bolt was assumed to carry the same bolt load and there appears to be no interaction effects between holes nor edge effects upon the stress distributions in the outside holes. The magnitudes and variations of the membrane forces and the effects of the two methods of representing the bolt transfer loads can thus be discussed with regard to either hole.

The largest radial force intensity (fig. 6) occurs, as might be expected, on the centerline of the bolt hole. The nonlinear bolt loading method predicts the largest radial membrane forces with a value of 32 kips/in. compression which is about 23 percent higher than the value based upon the cosine bolt loading. The largest circumferential membrane force (fig. 7) of 30 kips/in. tension occurs at an angle of about 75° from the bolt centerline and is also predicted by the nonlinear bolt loading technique.

This stress is about 15 percent higher than the value based upon the cosine bolt loading. The in-plane membrane shear forces (fig. 8) tend to be smaller than the radial or circumferential membrane forces. The largest membrane shear force is about 9 kips/in. and is due to the nonlinear bolt loading. Since the in-plane shearing forces tend to be small, the principal values (not shown) of the membrane forces correspond in magnitude and location to the maximum radial and circumferential membrane forces.

The distribution of the longitudinal membrane force N_x along the specimen centerline is shown in figure 9. At $x = 0$ the membrane force should be zero since this edge is stress free; the small nonzero value is indicative of the error in the finite element solution. The membrane force at the left edge of the bolt hole ($x = 0.5$) rises very sharply due to the indirect bearing load of the bolt. On the right side of the bolt hole, the force should also be zero since the bolt is not in contact at this point. The finite element solution tends to zero at this point. Away from the hole for increasing x, the membrane force approaches a uniform value given by the total applied force (21,816 lb), divided by the specimen width (3 in.).

Further insight into the results of the finite element analyses can be obtained by considering an elasticity solution for an isotropic medium. In reference 6, Bickley presents the plane stress elasticity solution for a hole in an infinite medium loaded by a cosine pressure distribution over one-half of the boundary of the hole. Closed form solutions for the stress components are given in polar coordinates in terms of the radius of the hole and Poisson's ratio. Tabulated data of the stress components for Poisson's ratio of 0.25 are also presented.

In figure 10 are shown the membrane force distributions predicted by Bickley for an infinite isotropic medium with a hole equal in radius to the bolt hole in the composite specimen and loaded by the bolt load used in the finite element analysis. The plots are made for $r/a = 1.2$ which corresponds closely to

laminated composite material was represented in NASTRAN as a homogeneous material with equivalent extensional stiffness. Laminate membrane force distributions were predicted.

Comparison of the two methods of representing the bolt transfer loads showed the two methods were in qualitative agreement. The nonlinear analysis estimated membrane forces about 20 to 25 percent higher than the linear analysis. Peak forces were found to be a radial compressive force on the bolt centerline and a circumferential tensile force of the same magnitude at about 70° from the centerline. In-plane shear forces were found to be relatively small. There were little or no interaction effects between holes or boundaries of the specimen. Comparison of the finite element solution with an isotropic elasticity solution suggests that as a rule these effects will not be important for in-plane membrane forces provided offset distances between holes or edges are greater than five hole radii.

REFERENCES

1. Advanced Composites Design Guide, Third Edition, Air Force Materials Laboratory, Wright-Patterson Air Force Base, Ohio, Chapter 1.3.

2. "Design and Fabrication of Mechanical Joint Specimens for Graphite-Epoxy Composites", by Hoffman, D.J. and June, R.R., Boeing Commercial Aircraft Company Report D6-41817, May 1974.

3. The NASTRAN Theoretical Manual, edited by R.H. MacNeal, NASA SP-221, September 1970, pp. 5.8-2 to 5.8-4.

4. Primer on Composite Materials: Analysis, by Ashton, J.E., Halpin, J.C., and Petit, P.H., Technomic Publishing Company, 1969.

5. "Stress and Deflection Analysis of Mechanically Fastened Joints", by Harris, H.G., Ojalvo, I.U., and Hooson, R.E. of Grumman Aerospace Corporation, Air Force Flight Dynamics Laboratory Report AFFDL-TR-70-49, May 1960.

6. "The Distribution of Stress Round a Circular Hole in a Plate", by Bickley, W.G., Phil. Trans. Royal Society (London) volume 227A, 1928, pp. 383-415.

Table 1. Extensional stiffnesses.

Section	$A_{ij} \times 10^{-6}$ (lb/in.)					
	A_{11}	A_{12}	A_{13}	A_{22}	A_{23}	A_{33}
1	3.01	0.932	0	1.34	0	0.984
2	2.94	0.877	-0.0492	1.27	-0.0492	0.927
3	2.87	0.823	0	1.20	0	0.870
4	2.65	0.816	0	1.17	0	0.861
5	2.43	0.809	0	1.15	0	0.853
6	2.36	0.755	0.0492	1.08	0.0492	0.796
7	2.28	0.699	0	1.01	0	0.739
8	2.07	0.693	0	0.986	0	0.730
9	2.00	0.639	-0.0492	0.914	-0.0492	0.673
10	1.92	0.584	0	0.841	0	0.616
11	1.85	0.529	-0.0492	0.769	-0.0492	0.560
12	1.78	0.474	0	0.697	0	0.503
13	1.71	0.419	-0.0492	0.625	-0.0492	0.446
14	1.63	0.364	0	0.553	0	0.389
15	1.42	0.358	0	0.531	0	0.380
16	1.34	0.303	0.0492	0.459	0.0492	0.323
17	1.13	0.297	0.0492	0.437	0.0492	0.314
18	1.05	0.242	0	0.365	0	0.258
19	1.05	0.242	0	0.365	0	0.258

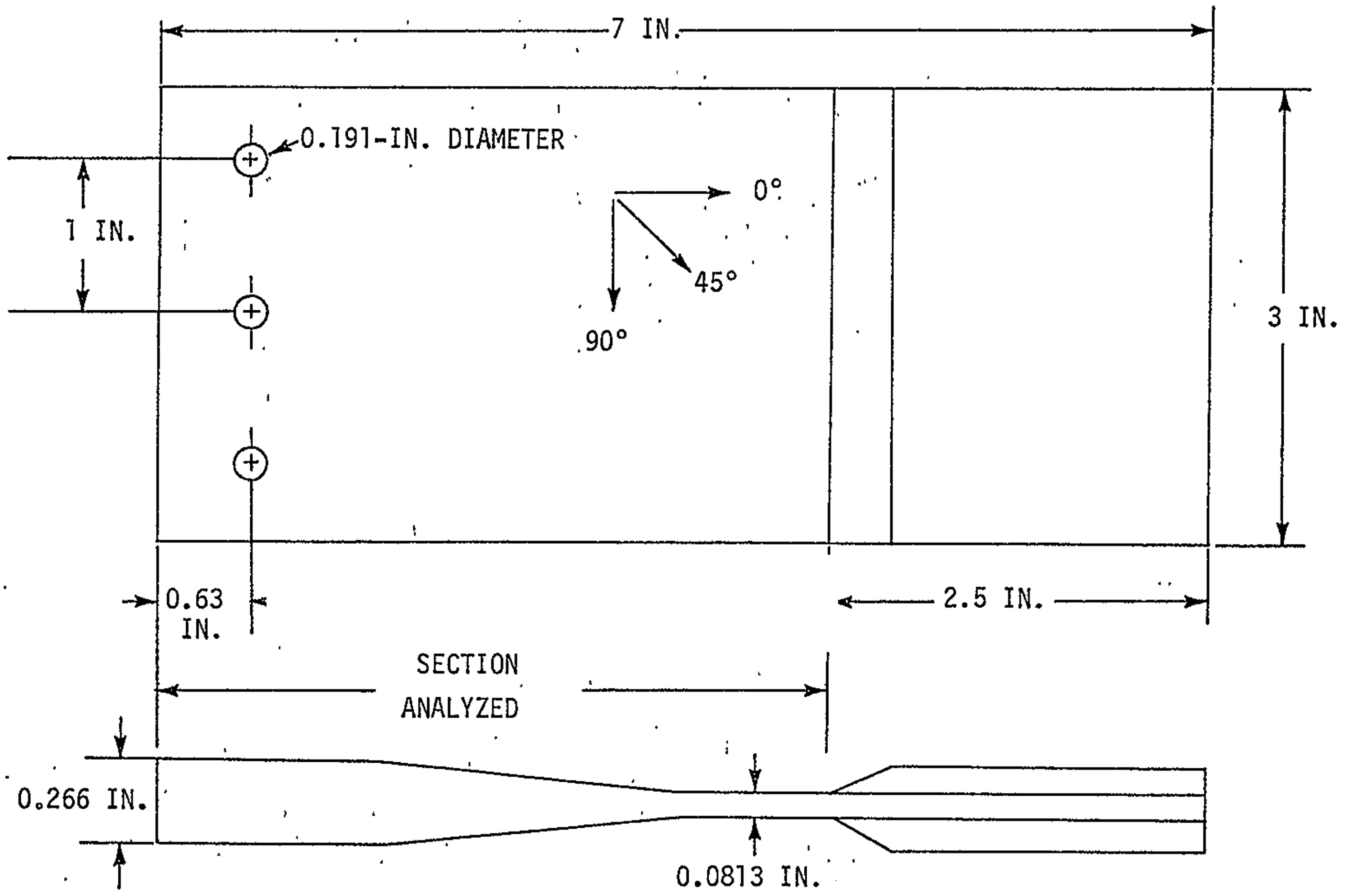


Figure 1. Extra Graphite Reinforced Joint Specimen.

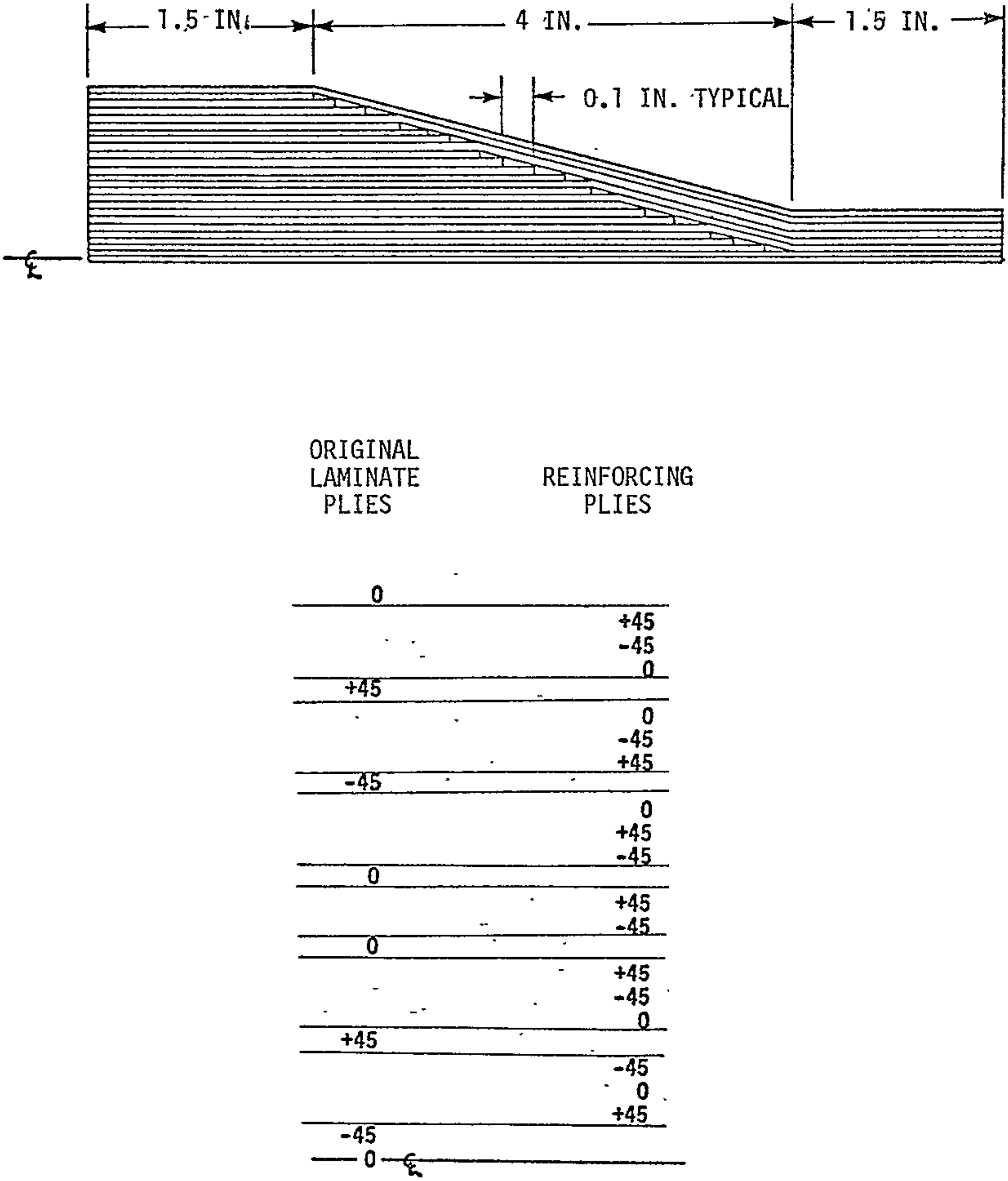


Figure 2. Ply Stacking Sequence for Graphite Reinforced Specimen 1.

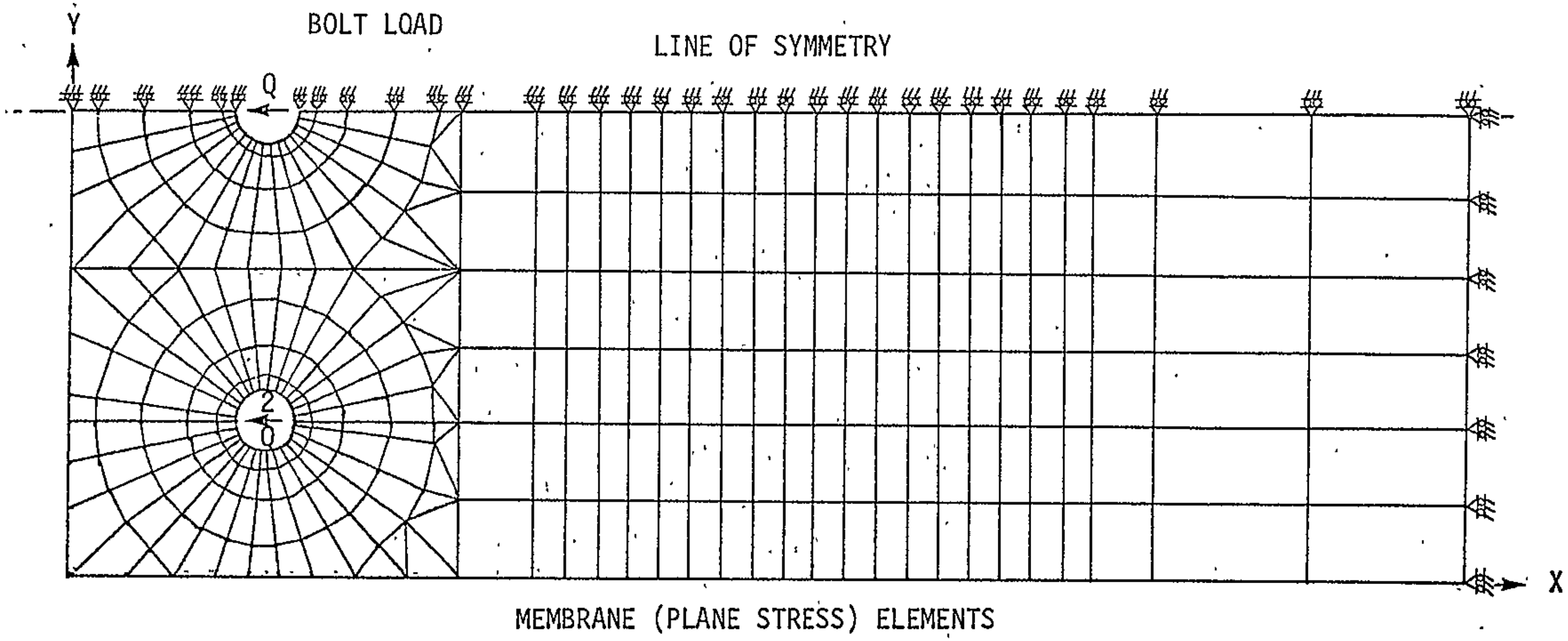


Figure 3. Finite Element Representation of Bolted Joint Specimen.

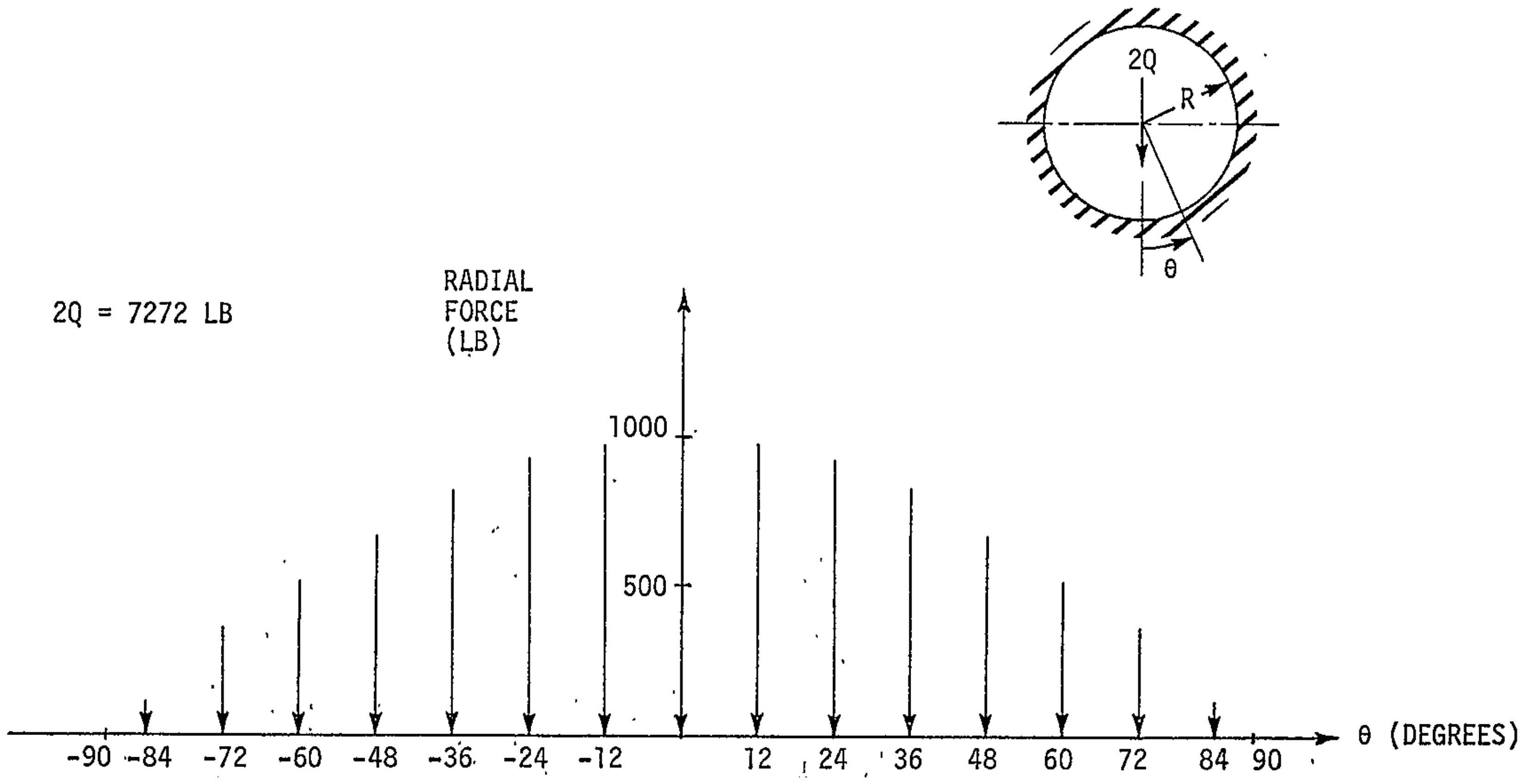


Figure 4. Bolt Transfer Loads for Cosine Load Distribution.

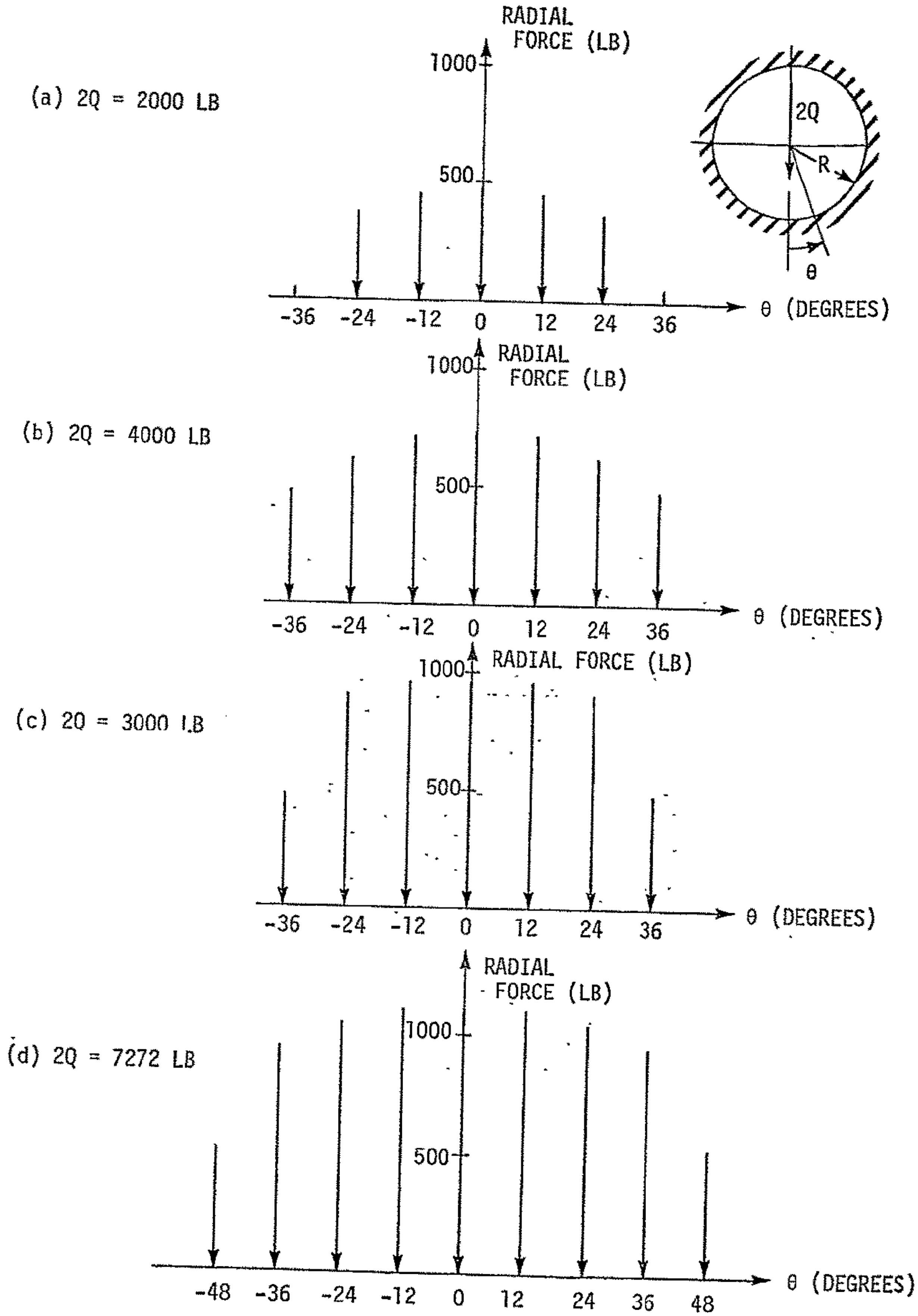


Figure 5. Bolt Transfer Loads from Nonlinear Loading Analysis.

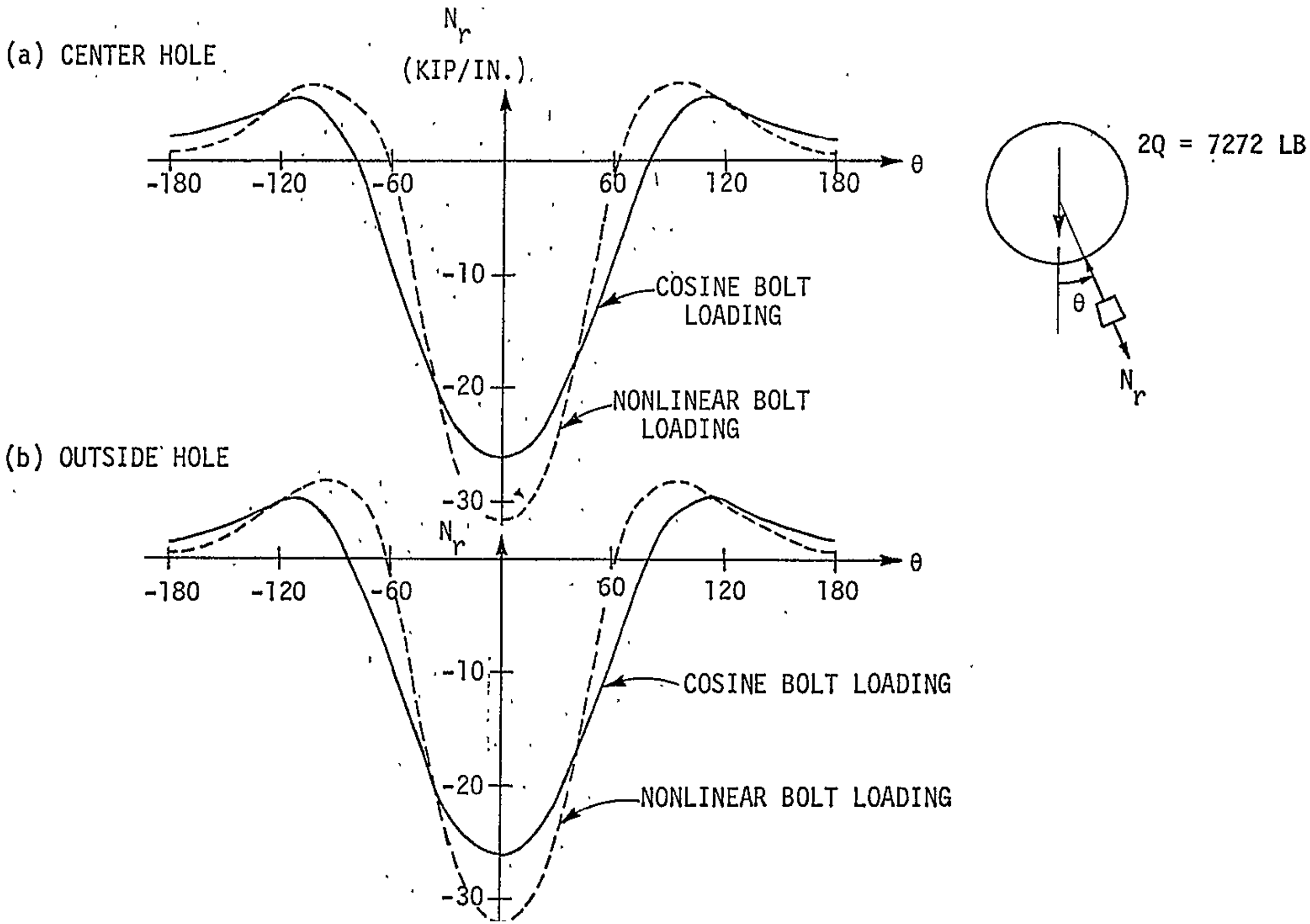


Figure 6. Radial Membrane Force Distributions at the Bolt Holes.

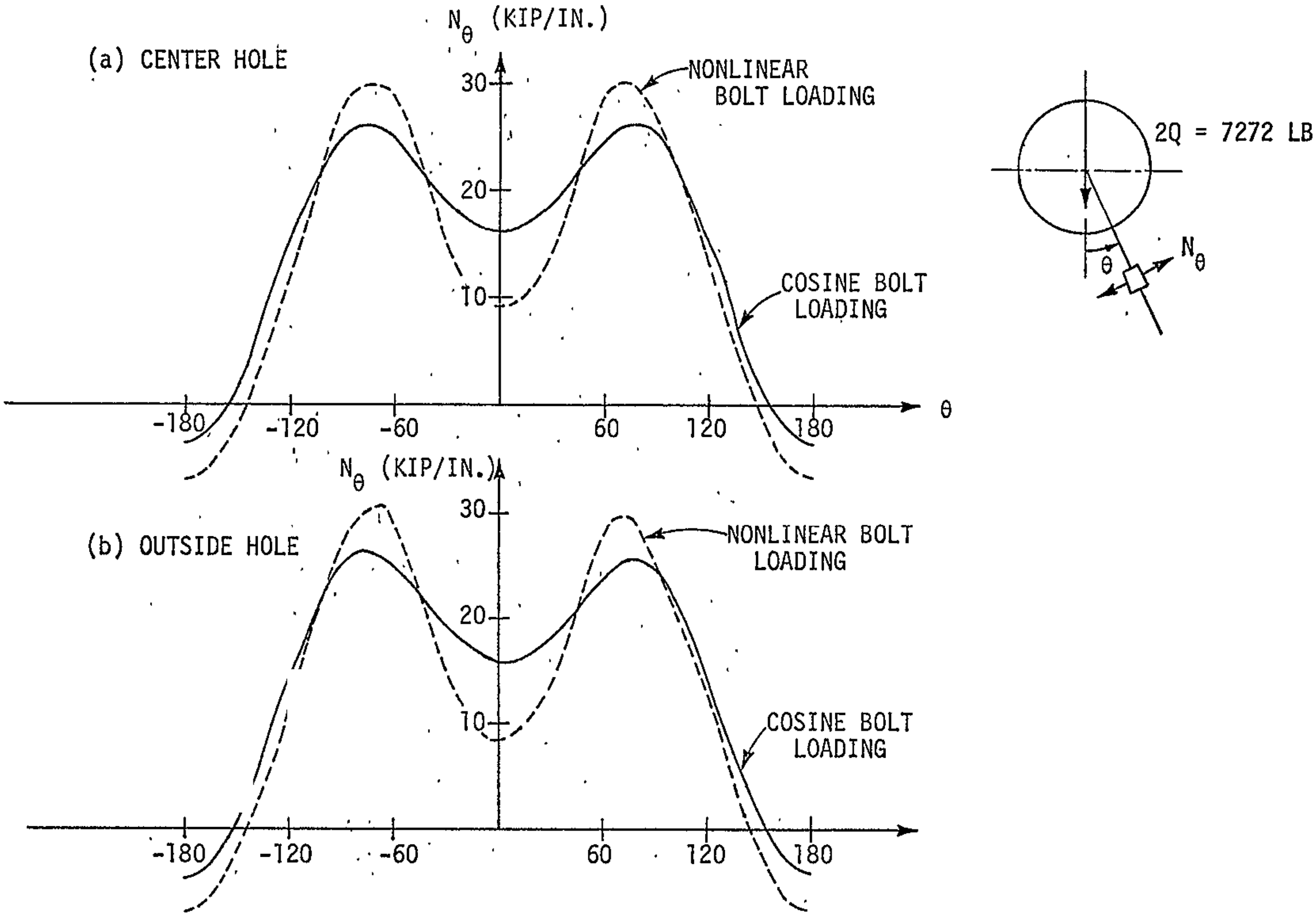


Figure 7. Circumferential Membrane Force Distributions at the Bolt Holes.

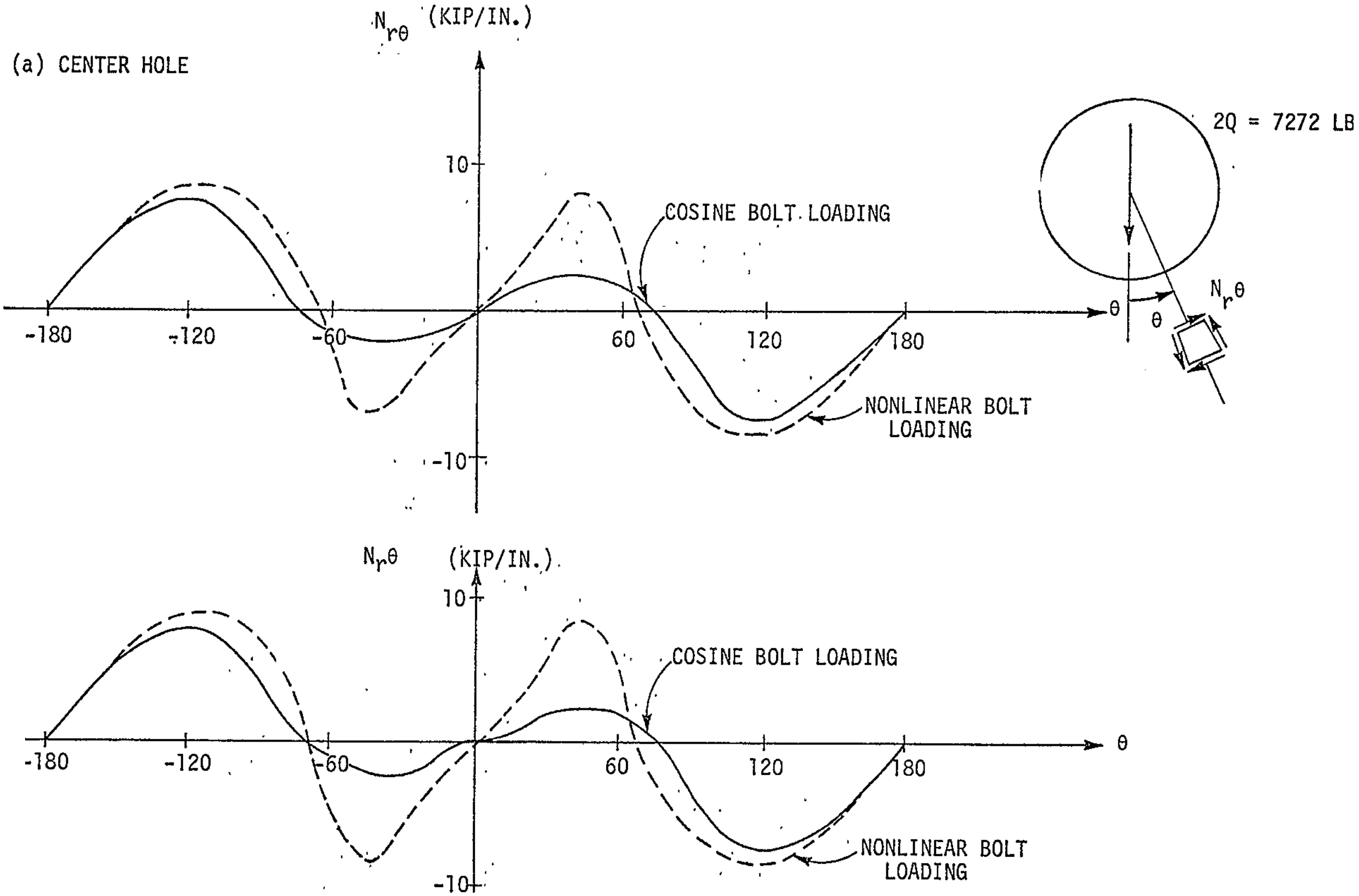


Figure 8. Membrane Shearing Force Distributions at the Bolt Hole.

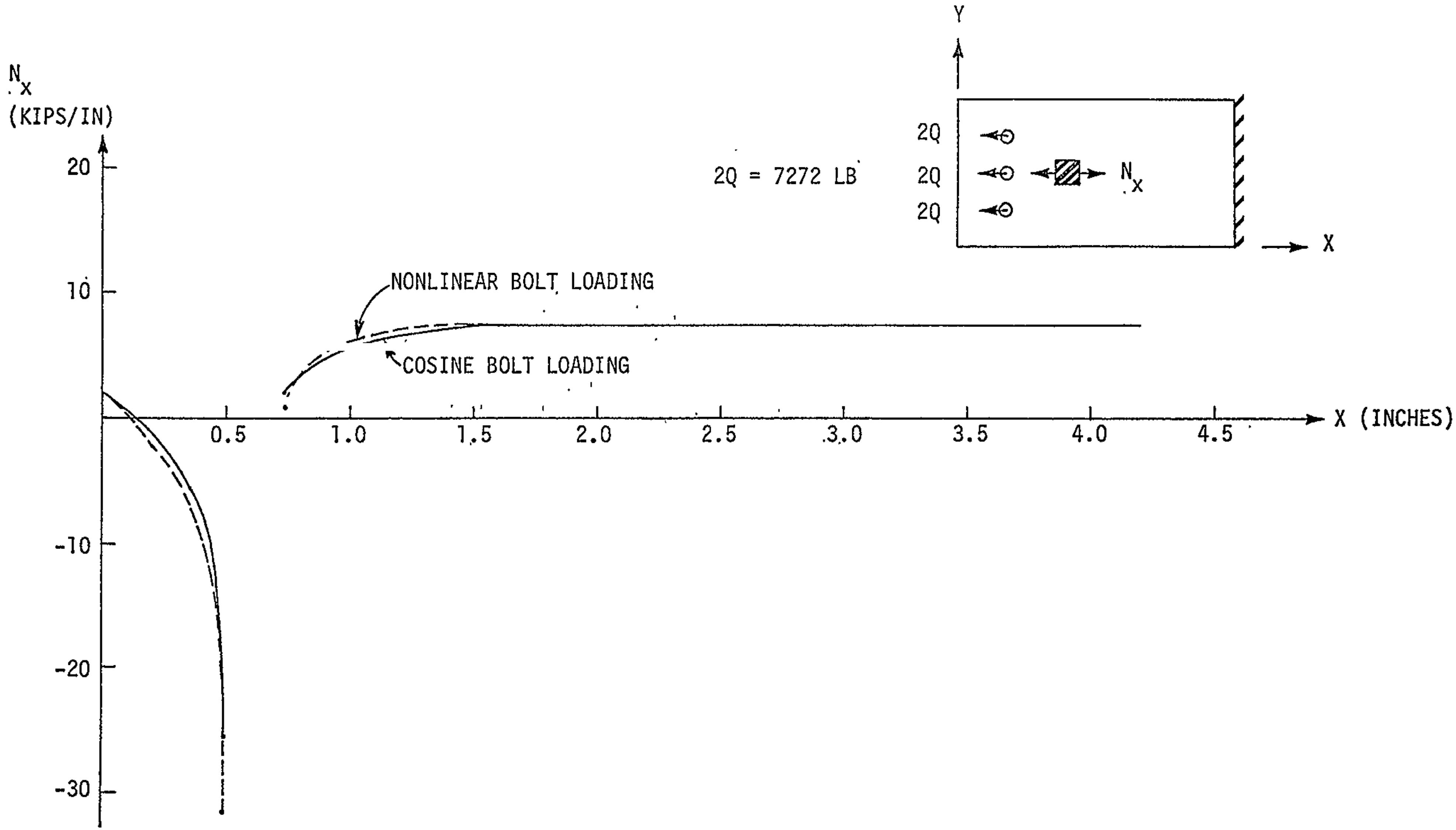
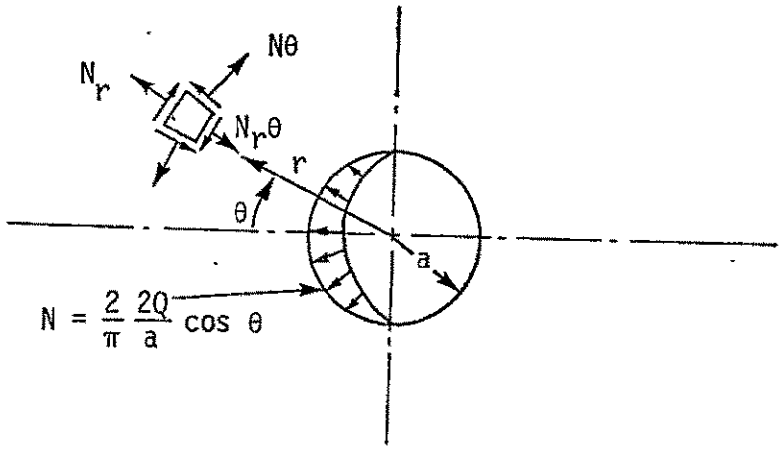
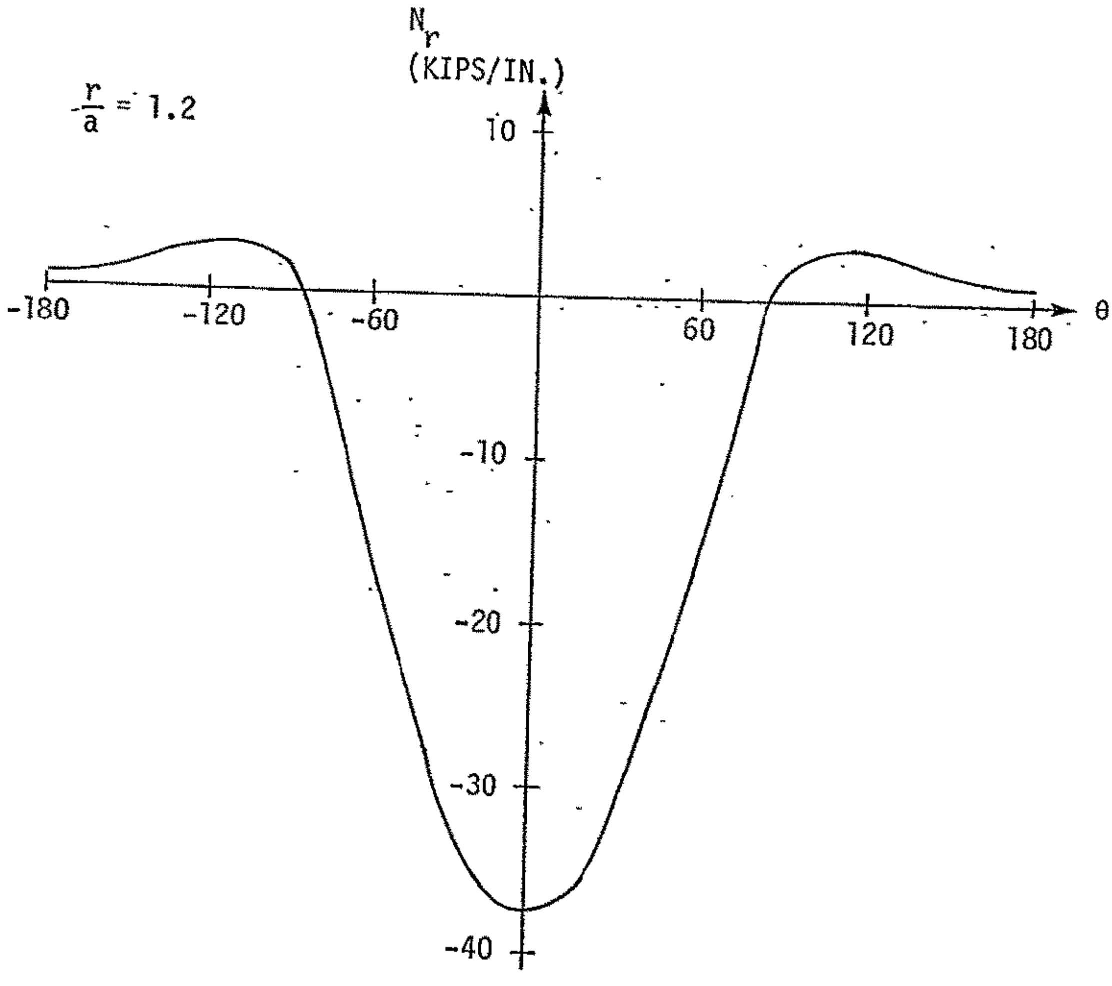


Figure 9. Longitudinal Membrane Force Distribution Along Specimen Centerline.

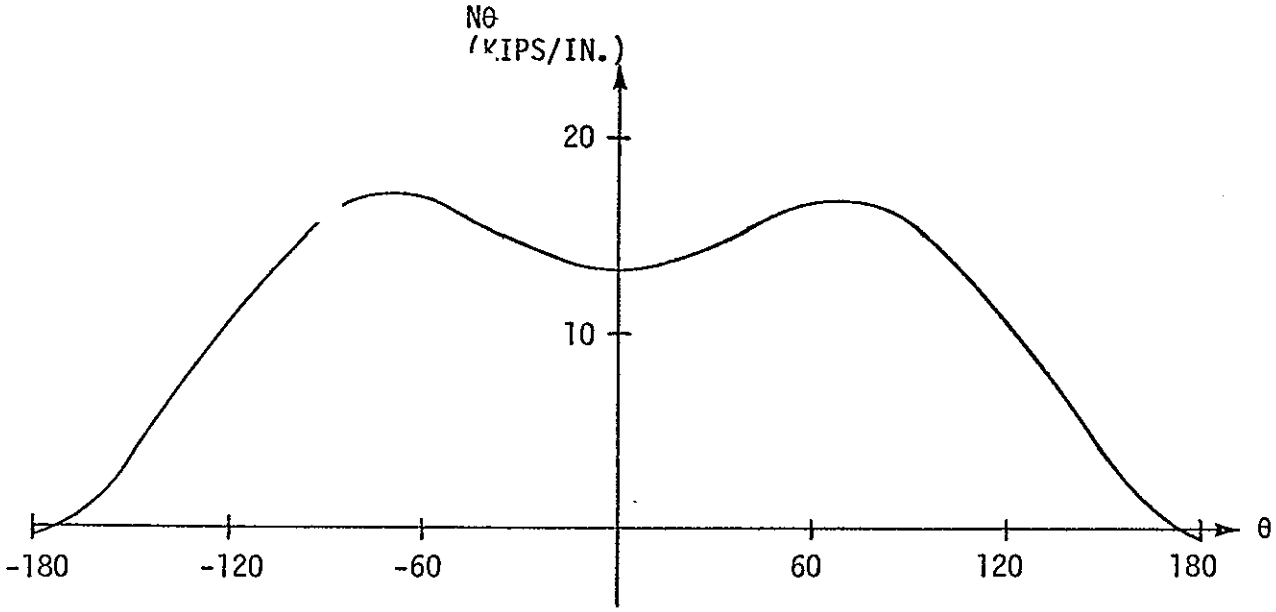


(a) INFINITE ISOTROPIC MEDIUM WITH COSINE BOLT LOADING

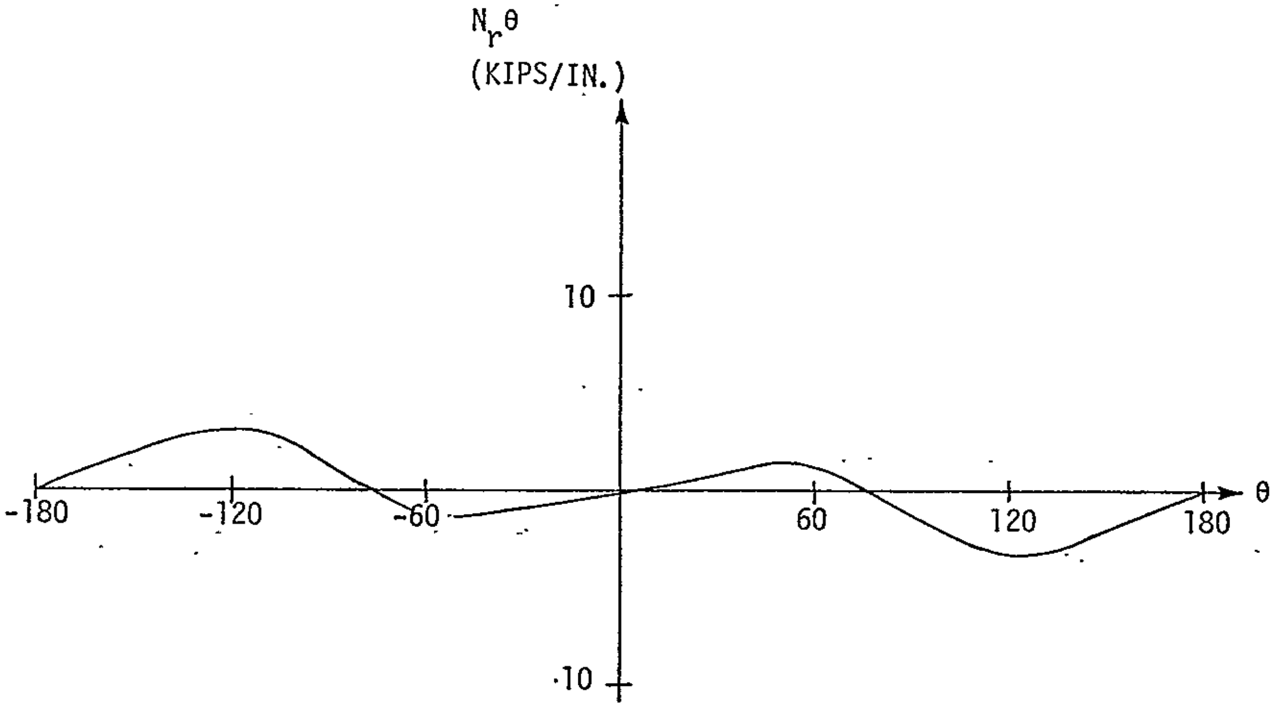


(b) RADIAL MEMBRANE FORCE DISTRIBUTION FOR r/a = 1.2.

Figure 10. Elasticity Solution for Infinite Isotropic Medium with Cosine Bolt Loading.



(c) CIRCUMFERENTIAL MEMBRANE FORCE DISTRIBUTION FOR r/a = 1.2.



(d) MEMBRANE SHEARING FORCE DISTRIBUTION FOR r/a = 1.2.

Figure 10 (concluded). Elasticity Solution for Infinite Isotropic Medium with Cosine Bolt Loading.

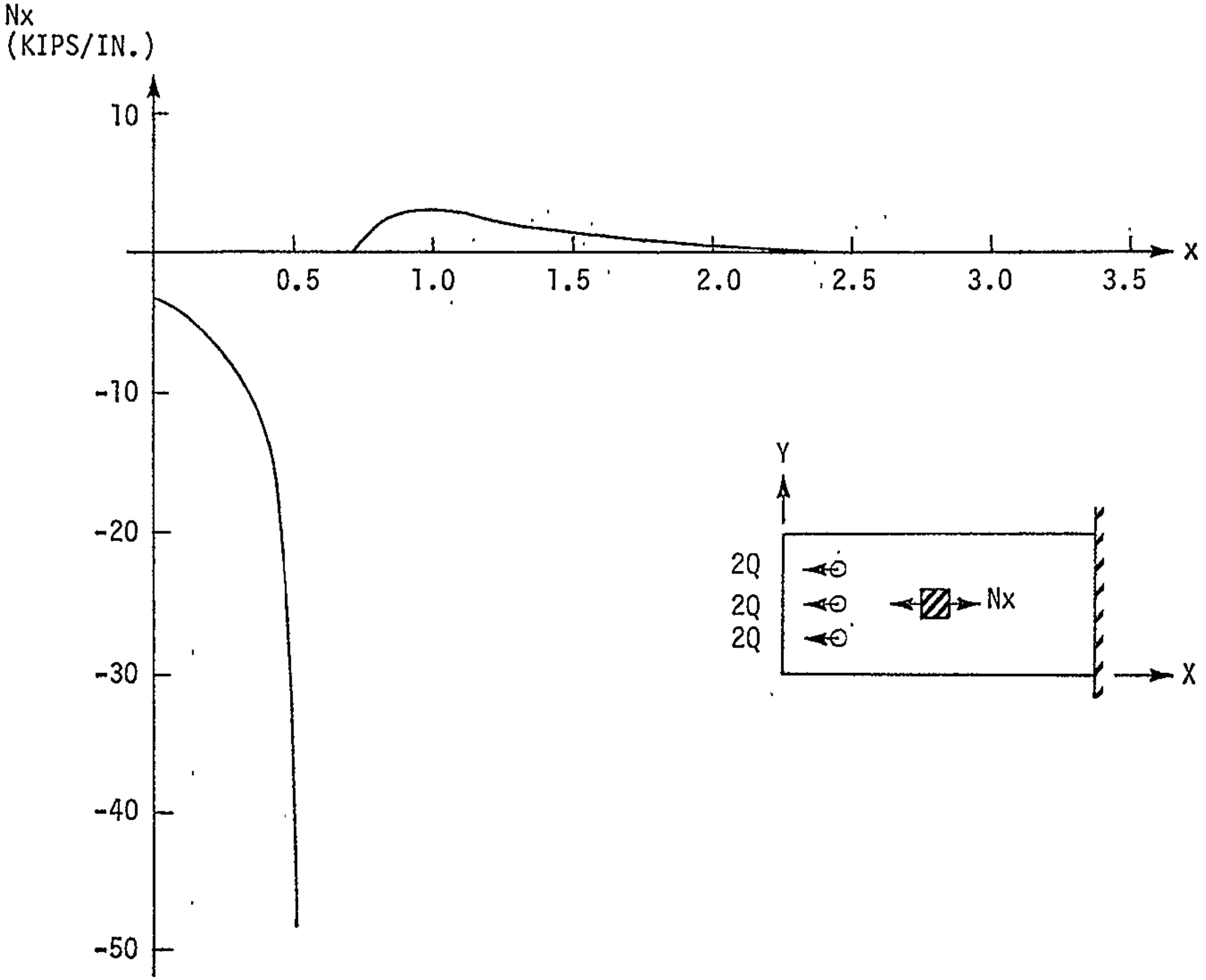


Figure 11. Elasticity Solution for Longitudinal Membrane Force Distribution Along Specimen Centerline.